

To estimate variation, the sample variance  $s^2$  is the best point estimator of the population variance  $\sigma^2$ . The sample variance is an unbiased estimator of the population variance which means that the values of the sample variance tend to center around the value of the population variance. The sample standard deviation,  $s$ , tends to underestimate the population standard deviation,  $\sigma$ , although the bias becomes smaller as the size of the sample increases.

### Example 10.1.3

#### Determining the Best Estimator of the Population Standard Deviation of Apple iPhone Costs

#### Technology

For instructions on how to compute the sample standard deviation using technology, please visit [stat.hawkeslearning.com](http://stat.hawkeslearning.com) and navigate to **Discovering Statistics and Data, Fourth Edition > Technology Instructions > Descriptive Statistics > One Variable**.

Using the data from Example 10.1.1, determine the best point estimate of the population standard deviation of iPhone costs.

#### Solution

The best estimator of the population standard deviation is the sample standard deviation. The sample standard deviation can be determined using technology or computed manually using the formula  $s = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n-1}}$  (see Section 4.2). The sample standard deviation using either method is 43.004. Therefore, the best estimate of the population standard deviation of iPhone costs is approximately \$43.

## 10.1 Exercises

### Basic Concepts

1. What is statistical inference?
2. What is an estimator?
3. Explain, in your own words, the difference between the terms *estimator* and *estimate*.
4. Give three examples of point estimators. Identify the parameters being estimated by these estimators.
5. What are two important questions to consider when estimating a population mean?
6. What is mean squared error?
7. What is an unbiased estimator? Give an example.
8. Why is the sample mean considered the best point estimate of the population mean?
9. Are all estimators unbiased? Explain.
10. What are two characteristics of the best available estimate for a parameter?
11. Are biased estimators useful? Give an example of one.

### Exercises

12. The mean monthly water bill for 79 randomly selected residents of the local apartment complex is \$138. What is the best point estimate for the mean monthly water bill for all residents of the local apartment complex?

13. At the local college, a study found that students earned an average of 8.8 credit hours per semester with a sample standard deviation of 2.8 credit hours. The study randomly selected a sample of 108 students. What is the best point estimate for the average number of credit hours per semester for all students at the local college?
14. Suppose a sample of 2500 new car buyers is drawn. Of those sampled, 882 preferred foreign cars over domestic cars. Determine a point estimate for the population proportion of new car buyers who prefer foreign cars over domestic.
15. An environmentalist draws a sample of 250 oil tankers. Thirty-two of the oil tankers had spills last year. Using this information, find a point estimate for the population proportion of oil tankers that had spills last year.
16. In a sample of 126 ChatGPT responses, the average response processing time was found to be 2.9 seconds with a variance of 1.2 seconds. Give a point estimate for the population standard deviation of ChatGPT response processing times.
17. Consider a sample of tires. Their diameters are measured and found to have a variance of 2.4 centimeters<sup>2</sup>. Give a point estimate for the population variance of tire diameters.
18. A sample of 6 bottles of sesame oil are randomly selected from a shipment of 3000 bottles. Each bottle is designed to contain 240 ml of sesame oil. The contents of bottles are measured, and the results are given below. Give a point estimate for the population standard deviation of the shipment.

240, 241, 235, 233, 234, 229

## 10.2 Estimating the Population Mean, $\sigma$ Known

Rarely will a point estimate of the population mean result in a value which exactly matches the population mean,  $\mu$ . If an estimate is used for decision making, it is desirable that there be some indication of its potential error. One of the significant limitations of simply reporting a point estimate is the lack of information concerning the estimator's accuracy.

Interval estimates, however, are constructed to provide additional information about the precision of the estimate. An **interval estimator** is made by developing an upper and a lower boundary for an interval that will hopefully contain the population parameter. It would be easy to construct an interval estimator that would always contain the population parameter: for example, the interval from negative infinity to positive infinity. But this particular estimator would not contain any useful information about the location of the population parameter. In interval estimation, the smaller the interval for a given level of confidence, the better the estimator.

### Interval Estimate

An **interval estimate** defines an upper and lower boundary for an interval that will hopefully contain the population parameter with a given confidence level.

DEFINITION

### Note

It is important to note that confidence and probability are conceptually related, but they are not the same thing.