Key Formulas

Section	
7.4	**Variance and Standard Deviation of a Binomial Random Variable** $\sigma^2 = V(X) = np(1-p)$ $\sigma = \sqrt{V(X)} = \sqrt{np(1-p)}$
7.5	**Poisson Probability Distribution Function** $P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$ where $e = 2.71828...$ and $\lambda =$ the mean number of successes.
7.6	**Hypergeometric Probability Distribution Function** $P(X = x) = \frac{{}_rC_x \; {}_{N-r}C_{n-x}}{{}_NC_n}$ where $r =$ the total number of successes possible, $N =$ the size of the total population, $n =$ the size of the sample drawn, $x =$ the number of successes in the sample of size n, and maximum of $(0, n+r-N) \le x \le$ minimum of (r, n). **Expected Value of a Hypergeometric Random Variable** $\mu = E(X) = n\left(\frac{r}{N}\right)$ **Variance of a Hypergeometric Random Variable** $\sigma^2 = V(X) = n\left(\frac{r}{N}\right)\left(1-\frac{r}{N}\right)\frac{(N-n)}{(N-1)}$.

Additional Exercises

1. Since 1930, the FIFA World Cup has been held nearly every four years with Qatar hosting in 2022. The following table shows the probability distribution for X, the goal differential for the World Cup Final.

Goal differential for the World Cup Final	0	1	2	3
Probability	$\frac{7}{20} = 0.35$	$\frac{4}{20} = 0.20$	$\frac{6}{20} = 0.30$	$\frac{3}{20} = 0.15$

a. What is the average goal differential in a World Cup Final?

b. Find the variance of the goal differential in a World Cup Final.

c. Find the standard deviation of the goal differential in a World Cup Final.

d. What is the probability that the goal differential will be 1 for a World Cup Final?

e. Find the probability that the goal differential will be at least 1 in a World Cup Final.

f. Find the probability that the goal differential will be at most 1 in a World Cup Final.

g. What is the most likely outcome of the World Cup Final according to the empirical data shown in the probability distribution table?

2. Consider the random variable X = outcome of rolling a 100-sided fair die. Then the possible values of X are $x = 1, 2, 3, \ldots 98, 99, 100$.

a. What type of probability distribution does X follow?

b. Describe the probability distribution of X both verbally and in mathematical form.

c. Determine the expected value and standard deviation of the random variable X.

d. Define another random variable Y as the numerical value of the outcome of rolling a 6-sided fair die. Compare the standard deviation of X to the standard deviation of Y. Which is larger? Explain why this is the case.

3. The US Department of Labor has issued a new set of guidelines governing certain work practices for employees. It estimates that only 20% of all firms will be subject to the new guidelines. To validate the estimate of the number of firms that will be affected by the new guidelines, the department randomly selects a sample of twenty firms for a study. Assuming their initial estimate of 20% is correct, answer the following questions.

a. What is the probability that 1 or fewer of the sampled firms will be subject to the new rules?

b. What is the probability that between 15 and 25 percent of the sampled firms will be subject to the rules?

c. One of the directors in the department remarked he thought that ten firms out of the sample would be subject to the rules. If the initial estimate is correct, what is the chance of this occurring?

4. Historically, the probability that a library book will be returned in one week is $p = 0.50$. The head librarian for the University Staff Hospital library is monitoring a random sample of 10 books to determine if the historical proportion of the books returned within one week, 0.50, has changed. Assuming the historical return rate is still the same, answer the following questions.

a. What is the probability that between four and six books will be returned in one week?

b. What is the chance that eight or more books will be returned in one week?

c. What is the probability that only one book will be returned in one week?

5. The number of fatalities resulting from automobile accidents for a 10-mile stretch of an interstate highway averages 1 per 100,000 automobiles. During a particular holiday weekend, 500,000 automobiles traveled over the 10-mile segment. Using a Poisson distribution, find the probability of each of the following.

a. No fatalities

b. 3 fatalities

c. At least one fatality

6. Compute the mean and variance for the following random variables.

a. The number of sixes obtained in 10 rolls of a single die.

b. The number of hearts in a 13-card bridge hand. (Draw 13 cards from a standard deck without replacement.)

c. The number of free throws made by a professional basketball player in his next 10 attempts. (Assume the player makes 88% of his free throws in the long run.)

d. The number of cracked eggs selected when randomly selecting 5 eggs from a 12-egg carton containing 2 cracked eggs.

e. The number of dots on the upper face when a single die is thrown.

7. A manufacturer of digital cameras knows that a shipment of 30 cameras sent to a large discount store contains three defective cameras. The manufacturer also knows that the store will choose two of the cameras at random, test them, and accept the shipment if neither one is defective.

a. Find the probability that at least one is defective.

b. What is the probability that the shipment is accepted?

8. On a roster of sixteen softball players, four are pitchers. Nine of the softball players are selected at random without replacement. What is the probability that exactly one of the nine players is a pitcher?

9. According to the American Hotel and Lodging Association (AH&LA), women accounted for 31% of business travelers last year.[2] Suppose that to attract these women business travelers, the AH&LA found that 80% of hotels offered thick, comfortable bath robes and towels. Consider a random and independent sample of 15 hotels.

a. Based on the information given, how many of the 15 hotels are expected to offer thick, comfortable bathrobes and towels?

b. Find the probability that all of the hotels in the sample offer thick, comfortable bathrobes and towels.

c. Find the probability that more than 5 but less than 9 of the hotels in the sample offer thick, comfortable bathrobes and towels.

10. A carnival has a game of chance: a fair coin is tossed. If it lands heads, you win \$1, and if it lands tails, you lose \$0.50. How much should a ticket cost to play this game if the carnival wants to break even?

11. You are working on a multiple-choice test which consists of 15 problems. Each of the problems has five answers, only one of which is correct. If you are totally unprepared for the test and are guessing, what is the probability that you don't get a zero on the test?

12. An automobile manufacturer is always trying to improve the quality of its vehicles. Assume that the number of defects per vehicle follows a Poisson distribution. If these defects occur randomly at an average rate of five per vehicle, what is the probability that a randomly selected vehicle will have at least one defect?

13. When proofreading a statistics textbook, one can expect to find a number of errors, whether they are typographical, symbolic, or even incorrect mathematical calculations. On average, a statistics textbook will contain 30 errors. What is the probability that when proofreading a text, one finds at least three errors? Assume that the number of errors found follows a Poisson distribution.

14. A jeweler was given a collection of twelve diamonds, of which three were synthetic (fake). If the jeweler selected two of these diamonds at random (without replacement), what is the probability that neither jewel is found to be synthetic?

15. L-Mart Inspections is a building inspection company. There were ten new commercial construction buildings completed in the last month and the sites are now available for inspection. L-Mart plans to inspect some of the new constructions for code violations and believes that half of the buildings will have violations.

a. What probability model would be appropriate for describing the number of buildings in the sample that have code violations? Explain your answer.

b. If L-Mart randomly selects four buildings to inspect, what is the probability that three of the buildings will have violations?

16. In the casino game of roulette, a wheel is spun and a ball is set in motion, ultimately coming to rest in one of the 38 slots on the wheel. Any slot is as likely as any other to capture the ball. Of the 38 slots, 18 are red, 18 are black, and 2 are green. Suppose the entry fee to play a single game is \$1 and the participant bets on red. If the ball comes to rest in one of the red slots, he wins \$1 in addition to getting back the original \$1 entry fee. If the ball does not end up in a red slot, the \$1 entry fee is lost. Let X denote the monetary gain when betting \$1 on red, in a single game of roulette. Gain is defined as the amount won minus the fee to play.

a. What are possible values of X?

b. Is X a discrete or a continuous random variable? Explain.

c. Construct the probability distribution of X.

d. Find the expected value of X and interpret this number.

e. Do you feel that in any casino games you would have a positive expected gain? Why?

17. An experiment consists of tossing two coins and a die simultaneously.

a. List the 24 equally likely simple events.

b. Define the random variable X as the sum of the number of heads on the two coins and the number of dots on the die. What are the possible values of X?

c. Construct the probability distribution of X in the form of a table.

d. Find the expected value of X.

18. A pediatric nurse is studying the number of babies born at a hospital with congenital defects. The nurse believes that the number of babies born during a month with congenital defects follows a Poisson distribution with an average of one baby born with a congenital defect per month.

a. Find the average number of babies born in one year with congenital defects.

b. Find the standard deviation of the number of babies born in one year with congenital defects.

c. Find the probability that no babies are born with congenital defects in one year.

d. Find the probability that at least 1 baby is born with a congenital defect in a year.

e. Find the probability that exactly 12 babies are born with a congenital defect in a year.

19. A lifelong beer drinker claims to be able to distinguish his favorite brand of beer, Sudriser, from all others. Two of his friends, doubtful of the claim, set up a taste test involving 4 brands of beer. Sudriser is poured into two mugs and each of the other 3 brands is also poured into 2 mugs, giving a total of 8 mugs of beer. The brand in each mug is written on a slip of paper taped to the bottom of the mug. The mugs are then arranged in a random order, and our friend is told to sample each mug and choose which 2 mugs contain Sudriser. Cheerfully, our friend complies.

Answer the following, assuming that the beer drinker really can't distinguish between beers.

a. What is the probability that the 2 mugs containing Sudriser are chosen?

b. What is the probability that at least one of the 2 mugs containing Sudriser is chosen?

c. What is the probability that the 2 mugs chosen contain the same brand?

d. What is the probability that the 2 mugs chosen contain different brands?