Key Formulas	
Section	
11.6	**Test Statistic for a Population Variance** $\chi^2 = \dfrac{(n-1)s^2}{\sigma_0^2}$ with $n-1$ degrees of freedom **Assumptions:** 1. The data is obtained via a random sample of size n. 2. The population is normally distributed with mean μ and standard deviation σ.

Additional Exercises

1. A tire company has found that the mean time required for a mechanic to replace a set of four tires is 62 minutes. After instituting a new installation procedure, the company believes that the expected time required to replace the set of four tires remains unchanged. A test of the company's belief will be performed.

a. What are the null and alternative hypotheses for the test of the company's belief?

b. Describe, in terms of the problem, how a Type I error could occur.

c. Describe, in terms of the problem, how a Type II error could occur.

2. Tech Transit wishes to test whether the mean number of passenger miles on a particular route exceeds 66,000 passenger miles, the number of passenger miles the company needs on that route to cover all allocated costs. A random sample of 25 trips on the route yields a mean of 70,250 miles and a standard deviation of 9000 miles. It is desired to control the significance level at 1%.

a. State the appropriate hypotheses for this problem.

b. Describe, in terms of the problem, how a Type I error could occur.

c. Describe, in terms of the problem, how a Type II error could occur.

3. A pain reliever currently being used in a hospital is known to bring relief to patients in a mean time of 3.5 minutes. To compare a new pain reliever with the one currently being used, the new drug is administered to a random sample of 50 patients. The mean time to relief for the sample of patients is 2.8 minutes and the standard deviation is 1.14 minutes. Does the data provide sufficient evidence to conclude that the new drug was effective in reducing the mean time until a patient receives relief from pain? Test using $\alpha = 0.10$.

4. Tech uses thousands of fluorescent light bulbs each year. The brand of bulb it currently uses has a mean life of 25,000 hours. A manufacturer claims that its new brand of bulbs, which cost the same as the brand the university currently uses, has a mean life of more than 25,000 hours. The university has decided to purchase the new brand if, when tested, the test evidence supports the manufacturer's claim at the 0.05 significance level. Suppose 64 bulbs were tested and they were found to have an average life of 25,550 hours. The population standard deviation is known to be 2000 hours. Will the university purchase the new brand of fluorescent bulbs?

5. The daily wages in a particular industry are normally distributed with a mean of \$13.20 and a standard deviation of \$2.50. If a company in this industry employing 40 workers pays these workers, on average, \$12.20, can this company be accused of paying inferior wages? Use a significance level of 1%.

6. A coin-operated soft drink machine was designed to discharge, on average, 12 ounces of beverage per cup. In a test of the machine, ten cupfuls of beverage were drawn from the machine and measured. The mean and standard deviation of the ten measurements were 12.1 ounces and 0.12 ounces, respectively. Do these data present sufficient evidence to indicate that the mean discharge differs from 12 ounces? Test using $\alpha = 0.10$.

7. Techside Real Estate, Inc. is a research firm that tracks the cost of apartment rentals in Southwest Virginia. Five years ago, the regional average apartment rental rate was \$895 per month. Assume that, based on the historical quarterly surveys, it is reasonable to assume that the population standard deviation is \$225. In a current study of apartment rental rates, a sample of 180 apartments in the region provided the apartment rental rates. Does the sample data enable Techside Real Estate, Inc. to conclude that the population mean apartment rental rate now exceeds the level reported five years ago? The sample mean is \$915 and the sample standard deviation is \$227.50. Make your decision based on $\alpha = 0.10$.

8. Suppose that the national average price for used cars is \$28,205. A manager of a local used car dealership reviewed a sample of 25 recent used car sales at the dealership in an attempt to determine whether the population mean price for the used cars at this particular dealership differed from the national mean. The prices for the sample of 25 cars are given in the data with a mean of \$27,750 and standard deviation of \$1400. Test using $\alpha = 0.05$ whether a difference exists in the mean price for used cars at the dealership.

9. Suppose you are responsible for auditing invoices. Historically, about 0.003 of the invoices possessed material errors. During the last audit cycle a number of suggestions were made and implemented, and you hope that the next audit will provide evidence of improvement in the error rate. An audit of 6000 recent invoices reveals 12 material errors.

a. Does the data suggest an improvement in the invoice error rate at $\alpha = 0.05$?

b. Compute the *P*-value of the test statistic.

c. Based on the *P*-value, would the decision change at $\alpha = 0.10$?

10. After completing Chemistry 101, a student decides to conduct an experiment on their favorite brand of whiskey to determine if the proof rating on the bottle is accurate. The student selects eight small eighty-proof bottles from different stores around town and measures the percent of alcohol in each bottle. (**Note:** 80-proof alcohol contains 40% alcohol.) The resulting measurements are as follows.

Percent of Alcohol per Bottle							
38%	40%	42%	41%	39%	38%	40%	38%

a. What is the population being studied?

b. What is the variable being measured?

c. What level of measurement does the data possess?

d. Can the student conclude that the actual proof of whiskey is not equal to 80 at $\alpha = 0.05$?

e. What assumption did the student make in performing the test in part **d.**?

11. You have decided to become a professional gambler specializing in roulette. If the roulette wheel is fair (each number has a $\frac{1}{38}$ chance) then you will lose in the long run. However, you plan to locate wheels that are not balanced properly. An unbalanced wheel will produce some numbers more often than expected. You believe that you have found such a wheel and have started keeping track of the number 29. After 420 spins of the wheel, the number 29 has been observed 14 times. Is this overwhelming evidence at the $\alpha = 0.05$ level that you should start betting heavily on the number 29?

12. A commercial airline is concerned over the increase in weight of carry-on luggage. In the past, the airline has estimated that the average piece of carry-on luggage will weigh 12 pounds. A random selection of 148 pieces of carry-on luggage has an average weight of 14.2 pounds with a standard deviation of 3.4 pounds. Do you think that the airline's concern is justified? Use $\alpha = 0.01$.

13. Consider the following hypothesis tests for the population mean with σ known. Compute the *P*-value for each test and decide whether you would reject or fail to reject the null hypothesis at $\alpha = 0.01$.

a. $H_0: \mu = 15, H_a: \mu > 15, z = 2.50$

b. $H_0: \mu = 80, H_a: \mu < 80, z = -1.95$

c. $H_0: \mu = 1200, H_a: \mu \neq 1200, z = 3.70$

14. Deli Delivery delivers sandwiches to neighboring office buildings during lunch time in New York City. The deli claims that the sandwiches will be delivered within 20 minutes from receiving the order. Given the hectic schedules of their customers, consistent delivery time is a must. The owner has decided that the standard deviation of delivery times should be at most 4 minutes. To determine how consistently the sandwiches are being delivered, the manager randomly selects 27 orders and measures the time from receiving the order to delivery of the sandwich. The average time to delivery of the sample was 20 minutes with a standard deviation of 4.5 minutes.

a. Will the manager conclude at $\alpha = 0.10$ that the delivery times vary more than the owner desires?

b. What assumption did you make about the delivery times in performing the test in part **a.**?

15. Consider the following hypothesis tests for the population mean with σ unknown. Compute the *P*-value for each of the tests and decide whether you would reject or fail to reject the null hypothesis at $\alpha = 0.05$.

a. $H_0: \mu = 12, H_a: \mu > 12, t = 1.75, n = 25$

b. $H_0: \mu = 0.12, H_a: \mu < 0.12, t = -2.95, n = 16$

c. $H_0: \mu = 55, H_a: \mu \neq 55, t = 2.35, n = 8$

16. In each of the following experimental situations, give the appropriate null and alternative hypotheses to be tested. Define all terms that appear in these hypotheses.

a. A random sample of 100 customers in a bank are selected and their times to be served are noted. The bank has recently retrained its tellers to be more efficient with the hope of decreasing its average time in servicing its customers, which has been 4 minutes in the past.

b. A local driver training school claims that at least 75% of its pupils pass the driving test on their first attempt. A sample of 60 students from the school are selected, and their performances on the driving test are noted. Based upon the data collected, we would like to refute the claim of the school.

c. A spokesperson for a popular diet claims that the average weight lost for someone on the diet will be at least 15 pounds over a two-month period. The amount of weight lost for each person in a sample of 10 people on the diet is determined in order to try to refute the claim of the diet spokesperson.

d. A tire company tests 68 of its new premium tires to determine if the average lifespan of the tire is more than the average lifespan of its major competitor's best tire. The average lifespan of the competitor's tire is 63,000 miles.

e. An elementary statistics student conducts an experiment in order to show that a coin from a magic kit is biased. The student flips the coin 500 times.

17. It is essential in the manufacture of machinery to utilize parts that conform to specifications. In the past, diameters of the ball bearings produced by a certain manufacturer had a variance of 0.00156. To cut costs, the manufacturer instituted a less expensive production method. The variance of the diameters of 101 randomly sampled bearings produced by the new process was 0.0021. Does the data provide sufficient evidence to indicate that the diameters of ball bearings produced by the new process are more variable than those produced by the old process? Test using $\alpha = 0.10$.

18. A national news magazine is interested in the proportion of counties in which the cost of living has decreased in the past 24 months. The news magazine believes that the true proportion is less than 30%. In a random sample of 100 counties, 20 counties had cost of living decreases.

a. Test the news magazine's claim at $\alpha = 0.08$.

b. Find the *P*-value for this test.

19. An increasing number of businesses are offering child-care benefits for their workers. However, one union claims that more than 90% of firms in the manufacturing sector still do not offer any child-care benefits to their workers. A random sample of 350 manufacturing firms is selected, and only 28 of them offer child-care benefits.

a. Does this sample result support the claim of the union? Test using $\alpha = 0.10$.

b. Calculate the *P*-value associated with this test.

20. During the holiday season, law enforcement officials estimated that 500 people would be killed and 25,000 injured on the nation's roads. They claimed that more than 50% of the accidents would be caused by drunk driving. A sample of 120 accidents showed that 67 were caused by drunk driving. Use this data to test their claim with $\alpha = 0.05$.

21. The quality control supervisor of a cannery is concerned about the variance of fill per can. Regulatory agencies specify that the standard deviation of the amount of fill should be less than 0.1 ounce. To determine whether the process is meeting this specification, the supervisor randomly selects ten cans, weighs the contents of each, and finds that the sample standard deviation of these measurements is 0.04. Do these data provide sufficient evidence to indicate that the variability is as small as desired? Test using $\alpha = 0.05$.