Formulas and Tables

Chapter 3

Class Width $= \dfrac{\text{maximum value} - \text{minimum value}}{\text{number of classes}}$

Relative Frequency $= \dfrac{\text{number in class}}{\text{total number of observations}}$

Chapter 4

Arithmetic Mean: $\dfrac{1}{n}(x_1 + x_2 + \ldots + x_n)$

Population Mean: $\mu = \dfrac{1}{N}(x_1 + x_2 + \ldots + x_N) = \dfrac{\sum x_i}{N}$

Sample Mean: $\bar{x} = \dfrac{1}{n}(x_1 + x_2 + \ldots + x_n) = \dfrac{\sum x_i}{n}$

Weighted Mean:

$$\bar{x} = \frac{w_1x_1 + w_2x_2 + \cdots + w_nx_n}{w_1 + w_2 + \cdots + w_n} = \frac{\sum(w_i \cdot x_i)}{\sum w_i}$$

Mean Absolute Deviation: $\text{MAD} = \dfrac{\sum|x_i - \bar{x}|}{n}$

Sample Variance: $s^2 = \dfrac{\sum(x_i - \bar{x})^2}{n-1}$

Population Variance: $\sigma^2 = \dfrac{\sum(x_i - \mu)^2}{N}$

Sample Standard Deviation: $s = \sqrt{s^2} = \sqrt{\dfrac{\sum(x_i - \bar{x})^2}{n-1}}$

Population Standard Deviation:

$$\sigma = \sqrt{\sigma^2} = \sqrt{\frac{\sum(x_i - \mu)^2}{N}}$$

Range = maximum data value − minimum data value

Location of the P^{th} percentile: $l = n\left(\dfrac{P}{100}\right)$

Percentile of x $= \dfrac{\text{number of data values} \leq x}{\text{total number of data values}} \cdot 100$

Q_1 **(location of 25th percentile):** $l = n\left(\dfrac{25}{100}\right)$

Q_2 **(location of 50th percentile):** $l = n\left(\dfrac{50}{100}\right)$

Q_3 **(location of 75th percentile):** $l = n\left(\dfrac{75}{100}\right)$

Interquartile Range $= Q_3 - Q_1$

***z*-Score:** $z = \dfrac{x - \mu}{\sigma}$

Outliers: Data value greater than $Q_3 + 1.5 \cdot \text{Interquartile Range}$ or less than $Q_1 - 1.5 \cdot \text{Interquartile Range}$.

Empirical Rule (Assumes the distribution of the data is bell-shaped):

One sigma rule: About 68% of the data should lie within one standard deviation of the mean.

Two sigma rule: About 95% of the data should lie within two standard deviations of the mean.

Three sigma rule: About 99.7% of the data should lie within three standard deviations of the mean.

Chebyshev's Theorem: The proportion of any data set lying within k standard deviations of the mean is at least $1 - \dfrac{1}{k^2}$, for $k > 1$.

Coefficient of Variation (for sample data): $CV = \left(\dfrac{s}{\bar{x}} \cdot 100\right)\%$

Coefficient of Variation (for a population): $CV = \left(\dfrac{\sigma}{\mu} \cdot 100\right)\%$

Mean of grouped data: $\mu = \dfrac{\sum f_i M_i}{N}$

Sample Mean of grouped data: $\bar{x} = \dfrac{\sum f_i M_i}{n}$

Variance of grouped data: $\sigma^2 = \dfrac{\sum(M_i - \mu)^2 f_i}{N}$

Sample Variance of grouped data: $s^2 = \dfrac{\sum(M_i - \bar{x})^2 f_i}{n-1}$

Midpoint $= \dfrac{\text{lowerclass boundary} + \text{upperclass boundary}}{2}$

Sample Proportion: $p = \dfrac{X}{n}$

Population Proportion: $p = \dfrac{X}{N}$

Chapter 5

Correlation Coefficient:

$$r = \frac{1}{n-1}\left\{\sum_{i=1}^{n}\left(\frac{x_i - \bar{x}}{s_x}\right)\left(\frac{y_i - \bar{y}}{s_y}\right)\right\}, \quad -1 \leq r \leq 1$$

Simple Linear Regression Model:

$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$

Residual (Error) = observed y − predicted y $= y_i - \hat{y}_i$

Estimated Simple Linear Regression Equation: $\hat{y} = b_0 + b_1 x_i$

Sum of Squared Errors (SSE):
$\text{SSE} = \Sigma \text{error}_i^2 = \Sigma\left(y_i - \hat{y}_i\right)^2 = \Sigma\left(y_i - \left(b_0 + b_1 x_i\right)\right)^2$

Slope (predicted): $b_1 = \dfrac{n\left(\Sigma x_i y_i\right) - \Sigma x_i \Sigma y_i}{n\left(\Sigma x_i^2\right) - \left(\Sigma x_i\right)^2}$

***y*-Intercept (predicted):**
$b_0 = \dfrac{1}{n}\left(\Sigma y_i - b_1 \Sigma x_i\right) = \overline{y} - b_1 \overline{x}$

Mean Square Error: $s_e^2 = \dfrac{\Sigma\left(y_i - \hat{y}_i\right)^2}{n-2} = \dfrac{\text{SSE}}{n-2}$

Standard Error: $s_e = \sqrt{s_e^2} = \sqrt{\dfrac{\Sigma\left(y_i - \hat{y}_i\right)^2}{n-2}} = \sqrt{\dfrac{\text{SSE}}{n-2}}$

Total Sum of Squares (Total SS): $\Sigma\left(y_i - \overline{y}\right)^2$

Sum of Squares of Regression (SSR): $\text{SSR} = \text{Total SS} - \text{SSE}$

Coefficient of Determination:

$$R^2 = \left(\frac{n\Sigma x_i y_i - \Sigma x_i \Sigma y_i}{\sqrt{\left[n\Sigma x_i^2 - \left(\Sigma x_i\right)^2\right]\left[n\Sigma y_i^2 - \left(\Sigma y_i\right)^2\right]}}\right)^2$$

$$= \frac{\text{SSR}}{\text{Total SS}} = 1 - \frac{\text{SSE}}{\text{Total SS}},\quad 0 \le R^2 \le 1$$

Chapter 6

Relative Frequency of *A*: $A = \dfrac{k}{n}$

Probability of Event A:
$P(A) = \dfrac{\text{number of outcomes in } A}{\text{total number of outcomes in the sample space}}$

Probability of a Complement: $P\left(A^c\right) = 1 - P(A)$

Odds: The odds in favor of an event A occurring is given by $\dfrac{P(A)}{P(\text{not } A)} = \dfrac{P(A)}{P\left(A^c\right)}$ The odds against an event A occurring is given by $\dfrac{P(\text{not } A)}{P(A)} = \dfrac{P\left(A^c\right)}{P(A)}$.

Union of Mutually Exclusive Events: $P(A \cup B) = P(A) + P(B)$

Addition Rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Conditional Probability: $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

Independent Events Rule:
$P(A \mid B) = P(A)$ and $P(B \mid A) = P(B)$

Multiplication Rule for Independent Events:
$P\left(A_1 \cap A_2 \cap \ldots \cap A_n\right) = P\left(A_1\right) \cdot P\left(A_2\right) \cdot \ldots \cdot P\left(A_n\right)$

Mulitplication Rule for Dependent Events:
$P(A \cap B) = P(A) \cdot P(B \mid A) = P(B) \cdot P(A \mid B)$

***n* Factorial:** $n! = n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 3 \cdot 2 \cdot 1$

Permutation: ${}_nP_k = \dfrac{n!}{(n-k)!}$

Distinguishable Permutation: given n_1 alike, n_2 alike,..., n_k alike, $\dfrac{n!}{\left(n_1! n_2! n_3! \ldots n_k!\right)}$

Combination: ${}_nC_k = \dfrac{n!}{(n-k)!k!}$

Bayes' Theorem:

$$P\left(B_i \middle| A\right) = \frac{P\left(A \middle| B_i\right) P\left(B_i\right)}{P\left(A \middle| B_1\right) P\left(B_1\right) + P\left(A \middle| B_2\right) P\left(B_2\right) + \cdots + P\left(A \middle| B_k\right) P\left(B_k\right)}$$

The Fundamental Counting Principle: If E_1 is an event with n_1 possible outcomes and E_2 is an event with n_2 possible outcomes, the number of ways the events can occur in sequence is $n_1 \cdot n_2$.

Chapter 7

Expected Value of a Discrete Random Variable: $\mu = E(X) = \Sigma\left[x \cdot p(x)\right]$

Variance of a Discrete Random Variable:
$\sigma^2 = V(X) = \Sigma(x-\mu)^2 \cdot p(x)$

Standard Deviation of a Discrete Random Variable: $\sigma = \sqrt{V(X)} = \sqrt{\Sigma(x-\mu)^2 \cdot p(x)}$

Binomial Probability Distribution:
$P(X = x) = {}_nC_x p^x (1-p)^{n-x}$

Expected Value of a Binomial Random Variable: $\mu = E(X) = np$

Variance of a Binomial Random Variable: $\sigma^2 = V(X) = np(1-p)$

Standard Deviation of a Binomial Random Variable: $\sigma = \sqrt{V(X)} = \sqrt{np(1-p)}$

Poisson Probability Distribution:

$$P(X=x)=\left(\frac{e^{-\lambda}\lambda^{x}}{x!}\right),\ \text{for } x=0,1,2,\ldots$$

Poisson Expected Value: $\mu=\lambda$

Poisson Variance: $\sigma^2=\lambda$

Poisson Standard Deviation: $\sigma=\sqrt{\sigma^2}=\sqrt{\lambda}$

Hypergeometric Probability Distribution:

$$P(X=x)=\frac{{}_{k}C_{x}\ {}_{N-k}C_{n-x}}{{}_{N}C_{n}},\ \text{where maximum of}$$

$(0, n+k-N) \le x \le$ minimum of (k,n)

Expected Value of a Hypergeometric Random Variable: $\propto = E(X)=n\cdot\frac{k}{N}$

Variance of a Hypergeometric Random Variable: $\sigma^2=V(X)=\left[n\cdot\frac{k}{N}\cdot\left(1-\frac{k}{N}\right)\cdot\frac{(N-n)}{(N-1)}\right]$

Chapter 8

Uniform Probability Density Function:

$$f(x)=\begin{cases}\dfrac{1}{b-a}, & \text{for } a\le x\le b\\ 0 & \text{otherwise}\end{cases}$$

Expected Value for a Continuous Uniform Random Variable: $\mu=E(x)=\frac{a+b}{2}$

Standard Deviation for a Continuous Uniform Random Variable: $\sigma=\frac{b-a}{\sqrt{12}}$

Standardizing a Normal Random Variable: $z=\frac{x-\mu}{\sigma}$

Chapter 9

Estimator of μ: An unbiased estimator of μ is $\bar{x}$.

Estimator of p: An unbiased estimator of p is p.

Estimator of σ^2: An unbiased estimator of σ^2 is s^2.

Variance of $\bar{x}$ (population of infinite size): $\sigma_{\bar{x}}^2=\frac{\sigma^2}{n}$

Variance of $\bar{x}$ (population of finite size): $\sigma_{\bar{x}}^2=\frac{(N-n)}{(N-1)}\cdot\frac{\sigma^2}{n}$

Central Limit Theorem – Mean of the Sample Means: $\mu_{\bar{x}}=E(\bar{x})=\mu$

Central Limit Theorem – Variance of the Sample Means: $\sigma_{\bar{x}}^2=\frac{\sigma^2}{n}$

Sample Proportion: $p=\frac{x}{n}$

Mean of the Sample Proportion (population of infinite size): $\mu_{p}=E\left(p\right)=p$

Variance of the Sample Proportion (population of infinite size): $\sigma_{p}^2=\frac{p(1-p)}{n}\approx\frac{p\left(1-p\right)}{n}$

Mean of the Sample Proportion (population of finite size): $\mu_{p}=E\left(p\right)=p$

Variance of the Sample Proportion (population of finite size):

$$\sigma_{p}^2=\frac{N-n}{N-1}\cdot\frac{p(1-p)}{n}\approx\frac{N-n}{N-1}\cdot\frac{p\left(1-p\right)}{n}$$

Chapter 10

***z*-Transformation:** $z=\frac{\bar{x}-\mu}{\sigma_{\bar{x}}}$

Mean Squared Error for the Sample Mean $\mathrm{MSE}(\bar{x})=E(\bar{x}-\mu)^2$

Confidence Interval for the Population Mean (σ is known): $\bar{x}\ \pm\ z_{\alpha/2}\ \frac{\sigma}{\sqrt{n}}$

***t*-test statistic:** $t=\dfrac{(\bar{x}\ -\ \mu)}{\frac{s}{\sqrt{n}}}$

Degrees of Freedom (*df*) for *t*-distribution:
$df\ =$ number of sample observations $-\ 1=n-1$

Confidence Interval for the Population Mean (σ is unknown): $\bar{x}\ \pm\ t_{\alpha/2,n-1}\ \frac{s}{\sqrt{n}}$

Critical Value for a *t*-distribution: $t_{\alpha/2,n-1}$

Minimum Sample Size to Select μ:

$$n=\left(\frac{z_{\alpha/2}\sigma}{error}\right)^2 \text{ (use } s \text{ if } \sigma \text{ is not known)}$$

Confidence Interval for the Population Proportion: $(np \geq 10,\ n(1-p) \geq 10)$:

$$p \pm z_{\alpha/2}\sigma_p\text{, where } \sigma_p=\sqrt{\frac{p(1-p)}{n}}$$

Minimum Sample Size to Estimate the Population Proportion: $n \approx \frac{z^2_{\alpha/2}p(1-p)}{error^2}$ (use $p=0.5$ if there is no estimate of p available)

Confidence Interval for the Population Variance:

$$\frac{(n-1)s^2}{\chi^2_{\alpha/2}} \leq \sigma^2 \leq \frac{(n-1)s^2}{\chi^2_{1-\alpha/2}}$$

Chapter 11

Hypothesis Test Statistic for population mean (σ is known): $z=\frac{\bar{x}-\mu_0}{\sigma_{\bar{x}}}$, where $\sigma_{\bar{x}}=\frac{\sigma}{\sqrt{n}}$

Hypothesis Test Statistic for population mean (σ is unknown): $t=\frac{\bar{x}-\mu_0}{s_{\bar{x}}}$, where $s_{\bar{x}}=\frac{s}{\sqrt{n}}$

Hypothesis Test Statistic for population proportion: $z=\frac{p-p}{\sigma_p}$, where $\sigma_p=\sqrt{\frac{p(1-p)}{n}}$

Hypothesis Test Statistic for a population variance:

$$\chi^2=\frac{(n-1)s^2}{\sigma_0^2} \text{ with degrees of freedom } = n-1$$

Chapter 12

Hypothesis Test Statistic and Confidence Interval (comparing two population means with σ_1 and σ_2 known):

$$z=\frac{(\bar{x}_1-\bar{x}_2)-\mu_{\bar{x}_1-\bar{x}_2}}{\sigma_{\bar{x}_1-\bar{x}_2}}=\frac{(\bar{x}_1-\bar{x}_2)-(\mu_1-\mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}}$$

$$(\bar{x}_1-\bar{x}_2) \pm z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}$$

Hypothesis Test Statistic and Confidence Interval (comparing two population means with σ_1 and σ_2 unknown and unequal):

$$(\bar{x}_1-\bar{x}_2) \pm t_{\alpha/2}\sqrt{\left(\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}\right)}$$

$$t=\frac{(\bar{x}_1-\bar{x}_2)-(\mu_1-\mu_2)}{\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}},$$

Hypothesis Test Statistic and Confidence Interval (comparing two population means with σ_1 and σ_2 unknown and assumed equal)

$$t=\frac{(\bar{x}_1-\bar{x}_2)-(\mu_1-\mu_2)}{\sqrt{s_p^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$$

$$(\bar{x}_1-\bar{x}_2) \pm t_{\alpha/2}\sqrt{s_p^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}$$

Pooled Sample Variance: $s_p^2=\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}$

Hypothesis Test Statistic and Confidence Interval (paired difference): $t=\frac{\bar{x}_d-\mu_d}{\frac{s_d}{\sqrt{n}}}$

$$\bar{x}_d \pm t_{\alpha/2}\frac{s_d}{\sqrt{n}}$$

Hypothesis Test Statistic and Confidence Interval (comparing two population proportions):

$$z=\frac{p_1-p_2-0}{\sqrt{\frac{\bar{p}(1-\bar{p})}{n_1}+\frac{\bar{p}(1-\bar{p})}{n_2}}}$$

$$(p_1-p_2) \pm z_{\alpha/2}\sqrt{\frac{p_1(1-p_1)}{n_1}+\frac{p_2(1-p_2)}{n_2}}$$

Pooled Sample Proportion: $\bar{p}=\frac{x_1+x_2}{n_1+n_2}$

Hypothesis Test Statistic and Confidence Interval (comparing two population variances): $F=\frac{s_1^2}{s_2^2}$.

$$\left(\frac{s_1^2}{s_2^2}\cdot\frac{1}{F_{\alpha/2}}\right)<\frac{\sigma_1^2}{\sigma_2^2}<\left(\frac{s_1^2}{s_2^2}\cdot\frac{1}{F_{(1-\alpha/2)}}\right)$$

Chapter 13

Simple Linear Regression Model: $y_i=\beta_0+\beta_1 x_i+\varepsilon_i$

Estimated Simple Linear Regression Equation: $y = b_0 + b_1 x$

Standard Deviation of the Sample Estimate

***b*₁:** $s_{b_1} = \sqrt{\dfrac{s_e^2}{\sum(x_i - \bar{x})^2}}$ (see Chapter 5 for s_e^2)

Confidence Interval for β_1: $b_1 \pm t_{\alpha/2, n-2} \cdot s_{b_1}$

Confidence Interval for the Average Value of *y* Given *x*: $\hat{y}_p \pm t_{\alpha/2, n-2} \cdot s_e \sqrt{\dfrac{1}{n} + \dfrac{(x_p - \bar{x})^2}{\sum(x_i - \bar{x})^2}}$

Confidence Interval for the Predicted Value of *y* Given *x*: $\hat{y}_p \pm t_{\alpha/2, n-2} \cdot s_e \sqrt{1 + \dfrac{1}{n} + \dfrac{(x_p - \bar{x})^2}{\sum(x_i - \bar{x})^2}}$

***t*-Statistic:** $t = \dfrac{b_1 - 0}{s_{b_1}} = \dfrac{b_1}{s_{b_1}}$

Chapter 14

Multiple Regression Model:

$$y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \ldots + \beta_k x_{ki} + \varepsilon_i$$

Estimated Multiple Regression Equation:

$$\hat{y}_i = b_0 + b_1 x_{1i} + b_2 x_{2i} + \cdots + b_k x_{ki}$$

F-Statistic: $F = \dfrac{\dfrac{SSR}{k}}{\dfrac{SSE}{n-(k+1)}} = \dfrac{\text{Mean Squared Regression}}{\text{Mean Squared Error}}$

Confidence Interval in a Multiple Regression Model: $b_i \pm t_{\alpha/2, n-(k+1)} s_{b_i}$, $i = 1, \ldots, k$

Adjusted *R*²: $R_a^2 = 1 - \left(\dfrac{n-1}{n-k-1}\right)\dfrac{\text{SSE}}{\text{Total SS}}$

Test Statistic for Testing the Hypothesis $\beta_i \neq 0$: $t = \dfrac{b_i - 0}{s_{b_i}} = \dfrac{b_i}{s_{b_i}} \quad i = 1, \ldots, k$

Chapter 15

Total Sum of Squares One-Way ANOVA:

Total SS = SSTreatment + SSError

$$\sum_{j=1}^{k}\sum_{i=1}^{n_j}\left(x_{ij} - \bar{\bar{x}}\right)^2 = \sum_{j=1}^{k} n_j \left(\bar{x}_j - \bar{\bar{x}}\right)^2 + \left\{\sum_{i=1}^{n_1}\left(x_{i1} - \bar{x}_1\right)^2 + \sum_{i=1}^{n_2}\left(x_{i2} - \bar{x}_2\right)^2 + \cdots + \sum_{i=1}^{n_k}\left(x_{ik} - \bar{x}_k\right)^2\right\}$$

***F*-Statistic One-Way ANOVA:**

$$F = \frac{\text{MST}}{\text{MSE}} = \frac{\dfrac{\sum_{j=1}^{k} n_j \left(\bar{x}_j - \bar{\bar{x}}\right)^2}{k-1}}{\dfrac{\sum_{i=1}^{n_1}\left(x_{i1} - \bar{x}_1\right)^2 + \sum_{i=1}^{n_2}\left(x_{i2} - \bar{x}_2\right)^2 + \cdots + \sum_{i=1}^{n_k}\left(x_{ik} - \bar{x}_k\right)^2}{N-k}}$$

Mean Square for Treatment (MST): MST = SST/(k–1)

$$\text{MST} = \frac{\dfrac{\left(\sum_{i=1}^{n_1} x_{i1}\right)^2}{n_1} + \dfrac{\left(\sum_{i=1}^{n_2} x_{i2}\right)^2}{n_2} + \cdots + \dfrac{\left(\sum_{i=1}^{n_k} x_{ik}\right)^2}{n_k} - \dfrac{\left(\sum_{j=1}^{k}\sum_{i=1}^{n_j} x_{ij}\right)^2}{N}}{k-1}$$

Mean Square for Error (MSE): MSE = SSE/(N–k)

$$\text{MSE} = \frac{\sum_{j=1}^{k}\sum_{i=1}^{n_j} x_{ij}^2 - \left\{\dfrac{\left(\sum_{i=1}^{n_1} x_{i1}\right)^2}{n_1} + \dfrac{\left(\sum_{i=1}^{n_2} x_{i2}\right)^2}{n_2} + \cdots + \dfrac{\left(\sum_{i=1}^{n_k} x_{ik}\right)^2}{n_k}\right\}}{N-k}$$

Fisher's Least Significant Difference Method

$$\left|\bar{x}_i - \bar{x}_j\right| \geq t_{\alpha/2, n_T - k}\sqrt{\text{MSE}\left(\frac{1}{n_i} + \frac{1}{n_j}\right)}$$

Fisher's LSD Method, Confidence Interval Approach

$$\left(\bar{x}_i - \bar{x}_j\right) \pm t_{\alpha/2, n_T - k}\sqrt{\text{MSE}\left(\frac{1}{n_i} + \frac{1}{n_j}\right)}$$

Tukey's Honest Significant Difference Method

$$\left|\bar{x}_i - \bar{x}_j\right| \geq q_{\alpha, k, n_T - k}\sqrt{\frac{\text{MSE}}{2}\left(\frac{1}{n_i} + \frac{1}{n_j}\right)}$$

Tukey's HSD Method, Confidence Interval Approach

$\left(\bar{x}_i - \bar{x}_j\right) \pm q_{\alpha, k, n_T - k}\sqrt{\dfrac{\text{MSE}}{n}}$ for **balanced data** ($n = n_i = n_j$)

or

$\left(\bar{x}_i - \bar{x}_j\right) \pm q_{\alpha, k, n_T - k}\sqrt{\dfrac{\text{MSE}}{2}\left(\dfrac{1}{n_i} + \dfrac{1}{n_j}\right)}$ for **unbalanced data** ($n_i \neq n_j$)

Sum of Squares Two-Way ANOVA (Randomized Block): Total SS = SSTreatment + SSBlock + SSError

***F*-Statistic for Two-Way ANOVA**

(Randomized Block): $F = \frac{\text{MST}}{\text{MSE}} = \frac{\frac{\text{SST}}{k-1}}{\frac{\text{SSE}}{(k-1)(b-1)}}$

F-Statistic for Blocking

$$= \frac{\text{MSBL}}{\text{MSE}} = \frac{\frac{\text{SSBL}}{b-1}}{\frac{\text{SSE}}{(k-1)(b-1)}}$$

Sum of Squares Two-way ANOVA (with Interaction): Total SS = SSFactorA +SSFactorB + SSInteraction +SSError

***F*-Statistic for Interaction between Factors:**

$$F = \frac{\frac{\text{SSAB}}{(a-1)(b-1)}}{\frac{\text{SSE}}{ab(r-1)}} = \frac{\text{MSAB}}{\text{MSE}}$$

***F*-Statistic for Main Effect for Factor A:**

$$F = \frac{\frac{\text{SSA}}{(a-1)}}{\frac{\text{SSE}}{ab(r-1)}} = \frac{\text{MSA}}{\text{MSE}}$$

***F*-Statistic for Main Effect for Factor B:**

$$F = \frac{\frac{\text{SSB}}{(b-1)}}{\frac{\text{SSE}}{ab(r-1)}} = \frac{\text{MSB}}{\text{MSE}}$$

Chapter 16

Chi-Square Test Statistic for Goodness of Fit:

$$\chi^2 = \sum_{i=1}^{k} \frac{\left[n_i - E(n_i)\right]^2}{E(n_i)}$$

Chi-Square Test Statistic for Association:

$$\chi^2 = \sum_{i=1}^{r} \sum_{j=1}^{c} \frac{\left[n_{ij} - E(n_{ij})\right]^2}{E(n_{ij})}$$

Chapter 17

Sign Test ($n \le 25$): X = the number of times the less frequent sign occurs

Sign Test ($n > 25$): $z = \frac{X + 0.5 - \left(\frac{n}{2}\right)}{\frac{\sqrt{n}}{2}}$

Wilcoxon Signed-Rank Test ($n \le 25$): H_a: >, then $T = T_+$ = the sum of the ranks of the positive differences; H_a: <, then $T = T_-$ = the sum of the ranks of the negative differences; H_a: $\neq$, then $T = \text{Min}(T_+, T_-)$.

Wilcoxon Signed-Rank Test ($n > 25$):

$$z = \frac{T - \frac{n(n+1)}{4}}{\sqrt{\frac{n(n+1)(2n+1)}{24}}}, \text{ where } T = \text{Min}(T_+, T_-)$$

Wilcoxon Rank-Sum Test ($n_1 \le 10$): $T = T_X$ = the rank sum of the smaller sample

Wilcoxon Rank-Sum Test ($n_1 > 10$):

$$z = \frac{T_X - \frac{n_1(n_1 + n_2 + 1)}{2}}{\sqrt{\frac{n_1 n_2 (n_1 + n_2 + 1)}{12}}}, \text{ where } T_X = \text{the}$$

rank sum of the smaller sample

Spearman Rank Correlation Coefficient:

$$r_s = 1 - \frac{6T}{n \cdot (n^2 - 1)}, \text{where } T = \sum_{i=1}^{n} [R(X_i) - R(Y_i)]^2$$

Rank Correlation Test ($n \le 30$): Reject null hypothesis if $|r_s| >$ critical value in Appendix A Table L.

Rank Correlation Test ($n > 30$): Reject null hypothesis if $|r_s| > \frac{z_{\alpha/2}}{\sqrt{n-1}}$

Runs Test for Randomness (α = 0.05, $m \le 20$, and $n \le 20$): R (number of runs)

Runs Test for Randomness (α = 0.05, $m > 20$, and $n > 20$): $z = \frac{(R - \mu_R)}{\sigma_R}$, where $\mu_R = 1 + \frac{2mn}{N}$,

$$\sigma_R = \sqrt{\frac{2mn(2mn - N)}{\left[N^2(N-1)\right]}}$$

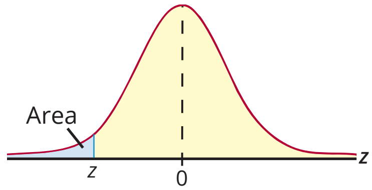
Kruskal-Wallis Test: $H = \frac{12}{N(N+1)} \sum_{i=1}^{k} \frac{R_i^2}{n_i} - 3(N+1)$,

where $N = \sum_{i=1}^{k} n_i$, $R_i = \sum_{j=1}^{n_i} r_{ij}$

Reject null hypothesis if $H \ge k_\alpha$, where value of k_α is found using the chi-square table in Appendix A Table G with $(k - 1)$ degrees of freedom and a predetermined value of α.

A Standard Normal Distribution

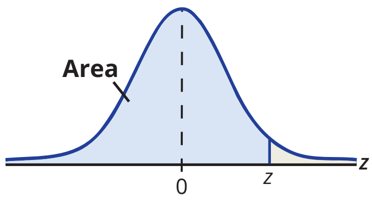
Numerical entries represent the probability that a standard normal random variable is between $-\infty$ and z where $z = \dfrac{x - \mu}{\sigma}$.



z	0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.02	0.01	0.00
–3.4	0.0002	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003
–3.3	0.0003	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0005	0.0005	0.0005
–3.2	0.0005	0.0005	0.0005	0.0006	0.0006	0.0006	0.0006	0.0006	0.0007	0.0007
–3.1	0.0007	0.0007	0.0008	0.0008	0.0008	0.0008	0.0009	0.0009	0.0009	0.0010
–3.0	0.0010	0.0010	0.0011	0.0011	0.0011	0.0012	0.0012	0.0013	0.0013	0.0013
–2.9	0.0014	0.0014	0.0015	0.0015	0.0016	0.0016	0.0017	0.0018	0.0018	0.0019
–2.8	0.0019	0.0020	0.0021	0.0021	0.0022	0.0023	0.0023	0.0024	0.0025	0.0026
–2.7	0.0026	0.0027	0.0028	0.0029	0.0030	0.0031	0.0032	0.0033	0.0034	0.0035
–2.6	0.0036	0.0037	0.0038	0.0039	0.0040	0.0041	0.0043	0.0044	0.0045	0.0047
–2.5	0.0048	0.0049	0.0051	0.0052	0.0054	0.0055	0.0057	0.0059	0.0060	0.0062
–2.4	0.0064	0.0066	0.0068	0.0069	0.0071	0.0073	0.0075	0.0078	0.0080	0.0082
–2.3	0.0084	0.0087	0.0089	0.0091	0.0094	0.0096	0.0099	0.0102	0.0104	0.0107
–2.2	0.0110	0.0113	0.0116	0.0119	0.0122	0.0125	0.0129	0.0132	0.0136	0.0139
–2.1	0.0143	0.0146	0.0150	0.0154	0.0158	0.0162	0.0166	0.0170	0.0174	0.0179
–2.0	0.0183	0.0188	0.0192	0.0197	0.0202	0.0207	0.0212	0.0217	0.0222	0.0228
–1.9	0.0233	0.0239	0.0244	0.0250	0.0256	0.0262	0.0268	0.0274	0.0281	0.0287
–1.8	0.0294	0.0301	0.0307	0.0314	0.0322	0.0329	0.0336	0.0344	0.0351	0.0359
–1.7	0.0367	0.0375	0.0384	0.0392	0.0401	0.0409	0.0418	0.0427	0.0436	0.0446
–1.6	0.0455	0.0465	0.0475	0.0485	0.0495	0.0505	0.0516	0.0526	0.0537	0.0548
–1.5	0.0559	0.0571	0.0582	0.0594	0.0606	0.0618	0.0630	0.0643	0.0655	0.0668
–1.4	0.0681	0.0694	0.0708	0.0721	0.0735	0.0749	0.0764	0.0778	0.0793	0.0808
–1.3	0.0823	0.0838	0.0853	0.0869	0.0885	0.0901	0.0918	0.0934	0.0951	0.0968
–1.2	0.0985	0.1003	0.1020	0.1038	0.1056	0.1075	0.1093	0.1112	0.1131	0.1151
–1.1	0.1170	0.1190	0.1210	0.1230	0.1251	0.1271	0.1292	0.1314	0.1335	0.1357
–1.0	0.1379	0.1401	0.1423	0.1446	0.1469	0.1492	0.1515	0.1539	0.1562	0.1587
–0.9	0.1611	0.1635	0.1660	0.1685	0.1711	0.1736	0.1762	0.1788	0.1814	0.1841
–0.8	0.1867	0.1894	0.1922	0.1949	0.1977	0.2005	0.2033	0.2061	0.2090	0.2119
–0.7	0.2148	0.2177	0.2206	0.2236	0.2266	0.2296	0.2327	0.2358	0.2389	0.2420
–0.6	0.2451	0.2483	0.2514	0.2546	0.2578	0.2611	0.2643	0.2676	0.2709	0.2743
–0.5	0.2776	0.2810	0.2843	0.2877	0.2912	0.2946	0.2981	0.3015	0.3050	0.3085
–0.4	0.3121	0.3156	0.3192	0.3228	0.3264	0.3300	0.3336	0.3372	0.3409	0.3446
–0.3	0.3483	0.3520	0.3557	0.3594	0.3632	0.3669	0.3707	0.3745	0.3783	0.3821
–0.2	0.3859	0.3897	0.3936	0.3974	0.4013	0.4052	0.4090	0.4129	0.4168	0.4207
–0.1	0.4247	0.4286	0.4325	0.4364	0.4404	0.4443	0.4483	0.4522	0.4562	0.4602
0.0	0.4641	0.4681	0.4721	0.4761	0.4801	0.4840	0.4880	0.4920	0.4960	0.5000

B Standard Normal Distribution

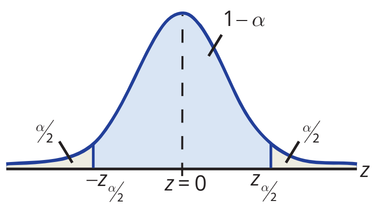
Numerical entries represent the probability that a standard normal random variable is between $-\infty$ and z where $z = \dfrac{x-\mu}{\sigma}$.



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

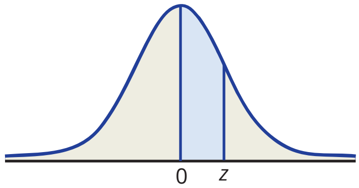
Critical Values

Level of Confidence	$z_{\alpha/2}$
0.80	1.28
0.90	1.645
0.95	1.96
0.98	2.33
0.99	2.575



C Standard Normal Distribution

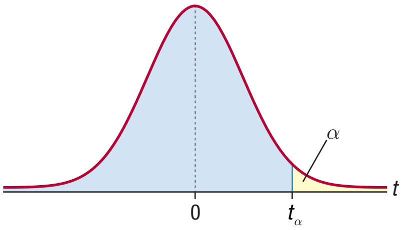
Numerical entries represent the probability that a standard normal random variable is between 0 and z where $z = \dfrac{x-\mu}{\sigma}$.



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998

D Critical Values of *t*

Numerical entries represent the value of t such that the area to the right of t is equal to α.



Degrees of Freedom	Area to the Right of the Critical Value					
	$t_{0.200}$	$t_{0.100}$	$t_{0.050}$	$t_{0.025}$	$t_{0.010}$	$t_{0.005}$
1	1.376	3.078	6.314	12.706	31.821	63.657
2	1.061	1.886	2.920	4.303	6.965	9.925
3	0.978	1.638	2.353	3.182	4.541	5.841
4	0.941	1.533	2.132	2.776	3.747	4.604
5	0.920	1.476	2.015	2.571	3.365	4.032
6	0.906	1.440	1.943	2.447	3.143	3.707
7	0.896	1.415	1.895	2.365	2.998	3.499
8	0.889	1.397	1.860	2.306	2.896	3.355
9	0.883	1.383	1.833	2.262	2.821	3.250
10	0.879	1.372	1.812	2.228	2.764	3.169
11	0.876	1.363	1.796	2.201	2.718	3.106
12	0.873	1.356	1.782	2.179	2.681	3.055
13	0.870	1.350	1.771	2.160	2.650	3.012
14	0.868	1.345	1.761	2.145	2.624	2.977
15	0.866	1.341	1.753	2.131	2.602	2.947
16	0.865	1.337	1.746	2.120	2.583	2.921
17	0.863	1.333	1.740	2.110	2.567	2.898
18	0.862	1.330	1.734	2.101	2.552	2.878
19	0.861	1.328	1.729	2.093	2.539	2.861
20	0.860	1.325	1.725	2.086	2.528	2.845
21	0.859	1.323	1.721	2.080	2.518	2.831
22	0.858	1.321	1.717	2.074	2.508	2.819
23	0.858	1.319	1.714	2.069	2.500	2.807
24	0.857	1.318	1.711	2.064	2.492	2.797
25	0.856	1.316	1.708	2.060	2.485	2.787
26	0.856	1.315	1.706	2.056	2.479	2.779
27	0.855	1.314	1.703	2.052	2.473	2.771
28	0.855	1.313	1.701	2.048	2.467	2.763
29	0.854	1.311	1.699	2.045	2.462	2.756
30	0.854	1.310	1.697	2.042	2.457	2.750
40	0.851	1.303	1.684	2.021	2.423	2.704
50	0.849	1.299	1.676	2.009	2.403	2.678
60	0.848	1.296	1.671	2.000	2.390	2.660
70	0.847	1.294	1.667	1.994	2.381	2.648
80	0.846	1.292	1.664	1.990	2.374	2.639
90	0.846	1.291	1.662	1.987	2.368	2.632
100	0.845	1.290	1.660	1.984	2.364	2.626
120	0.845	1.289	1.658	1.980	2.358	2.617
∞	0.842	1.282	1.645	1.96	2.326	2.576