

Chapter Project

Darts and the Normal Distribution

Darts, like any game of skill, involves some degree of randomness. In this project, we will try to analyze the chances of earning various point totals in individual dart tosses. A typical dartboard is displayed in Figure 1.

We may consider two random variables in the position of each dart toss (see Figure 2):

1. The angle, θ , from the horizontal of the line (in red) on which the dart lands.
2. The position along this line, X , centered at the middle of the bullseye.

If the location of the dart is the black dot in Figure 2, we see that X and θ define the location of the dart, with a positive X above the horizontal and a negative X below it.

If a player aims a dart at the bullseye in the center of the dartboard and tosses it, of course he/she may or may not hit the target with some probability, but we can say more if we know something about the pattern in the randomness.

In particular, a highly skilled player when aiming at the bullseye, will hit the center of the bullseye on average, but with decreasing probability, the dart will land further away from the target, so a normal distribution centered at 0 is reasonable for X . A highly skilled player also shows no particular angular tendency, so we may model θ as a uniformly distributed random variable that is independent of X .

Common rules of darts have the following scoring scheme¹:

Where dart lands	Point value
Inner Bullseye	50
Outer Bullseye	25
Single Region (black/white area within Double ring)	Score given on outside ring (1-20)
Triple Ring (inner red/green ring)	Triple the score given on the outside ring
Double Ring (outer red/green ring)	Double the score given on the outside ring
Outside the Double Ring	0



Figure 1

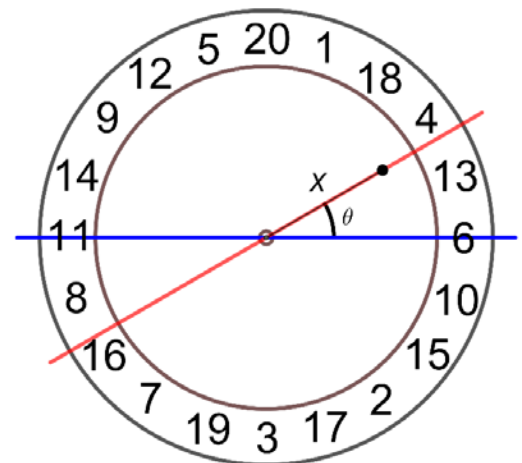


Figure 2

Questions (Assume all throws are aimed at the center of the bullseye.)

1. What is the range of the random variable θ ?
2. For a skilled player, what is the mean of the normal random variable X ?
3. Should the normal random variable X have a high or low σ for the most skilled players?
4. Find the probability of the dart landing in each region for a player with $\sigma = 50$ mm and for a player with $\sigma = 100$ mm given the following specifications for the dartboard. Each region corresponds to the given ranges of (absolute) distances from the center of the inner bullseye.¹

Region	Distance is more than...	Distance is less than...
Inner Bullseye	0 mm	6.35 mm
Outer Bullseye	6.35 mm	15.9 mm
Inner Single Region	15.9 mm	95.3 mm
Triple Ring	95.3 mm	104.8 mm
Outer Single Region	104.8 mm	158.8 mm
Double Ring	158.8 mm	168.3 mm
Beyond	168.3 mm	

5. What is the probability of landing in a single scoring region (shown in yellow in Figure 3) for each player?
6. What is the probability of landing in the wedge of the dartboard marked with 20 if $\sigma = 50$ mm (i.e., in the yellow region or beyond)? (See Figure 4.)
7. What is the probability of scoring 7 points on one dart throw if $\sigma = 50$ mm?
8. What is the probability of scoring 8 points on one dart throw if $\sigma = 50$ mm?
9. What is the probability of scoring 6 points on one dart throw if $\sigma = 50$ mm?
10. What are the highest and lowest possible scores from a single dart throw? What is the probability of each if $\sigma = 50$ mm?
11. What is the expected value of the score for a dart toss by a skilled player if $\sigma = 50$ mm?

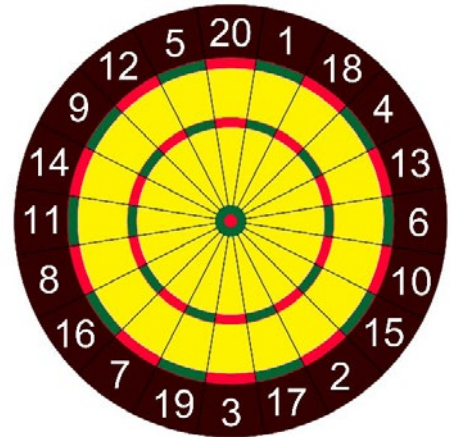


Figure 3



Figure 4

References

¹World Darts Federation (2018). Playing and Tournament Rules. 20th edition. <https://www.dartswdf.com/rules/>