Completion Example 12 Finding Equivalent Fractions

Find the missing numerator that will make the fractions equivalent.

$$\frac{3}{5} = \frac{?}{55}$$

Solution

Because $5 \cdot \_\_\_ = 55$, we have $\frac{3}{5} = \frac{3}{5} \cdot \frac{\_\_\_\_\_}{\_\_\_\_\_}$. Thus, $\frac{3}{5} = \frac{\_\_\_\_\_}{55}$.

Now work margin exercise 12.

12. Find the missing numerator that will make the fractions equivalent.

$$\frac{4}{7} = \frac{?}{63}$$

Completion Example Answers

3. $2 \cdot 2 \cdot 3 \cdot 3$; $2 \cdot 2 \cdot 2 \cdot 3$; $2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$; 2 and 3; Four 2s; Two 3s; LCM $= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 = 144$; 144

8. $30 = 2 \cdot 3 \cdot 5$, $50 = 2 \cdot 5 \cdot 5$, $63 = 3 \cdot 3 \cdot 7$; LCM $= 2 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 7 = 3150$;

$3150 = 2 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 7 = (2 \cdot 3 \cdot 5)(3 \cdot 5 \cdot 7) = 30 \cdot 105$; 105 times;

$3150 = 2 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 7 = (2 \cdot 5 \cdot 5)(3 \cdot 3 \cdot 7) = 50 \cdot 63$, 63 times;

$3150 = 2 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 7 = (3 \cdot 3 \cdot 7)(2 \cdot 5 \cdot 5) = 63 \cdot 50$, 50 times

12. $5 \cdot 11$; $\frac{3}{5} = \frac{3}{5} \cdot \frac{11}{11} = \frac{33}{55}$; $\frac{3}{5} = \frac{33}{55}$

Margin Exercise Answers

1. 84 **2.** 126 **3.** 120 **4.** 180 **5.** 441 **6.** LCM = 210; $210 = 15 \cdot 14$; $210 = 35 \cdot 6$; $210 = 42 \cdot 5$
7. LCM = 450; $450 = 10 \cdot 45$; $450 = 18 \cdot 25$; $450 = 75 \cdot 6$ **8.** LCM = 900; $900 = 20 \cdot 45$; $900 = 25 \cdot 36$; $900 = 36 \cdot 25$ **9. a.** 30 minutes **b.** 5, 10, and 6 laps **10.** 8 **11.** 15 **12.** 36

2.5 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. 5, 10, 15, 20, 25, … are _____ of the number 5.

2. LCM stands for _____ _____ _____.

3. To find the LCM of a set of counting numbers, the first step is to find the _____ _____ of each number.

4. The _____ of a number are the products of that number with the counting numbers.

5. The LCM of 20 and 25 is _____.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

6. The LCM of 15 and 25 is 50.

7. The first five multiples of 9 are 9, 18, 27, 36, and 45.

8. The first five multiples of 4 are 4, 8, 12, 20, and 24.

9. When given larger numbers, the most efficient way to find the LCM is to use the prime factorization method.

Practice

List the first twelve multiples of each number.

1. 5

2. 6

3. 12

4. 15

From the lists you made in Exercises 1 through 4, find the least common multiple for the following pairs of numbers.

5. 5 and 6

6. 5 and 12

7. 5 and 15

8. 6 and 12

9. 6 and 15

10. 12 and 15

Find the LCM of each set of numbers. See Examples 1 through 5.

11. 6, 10

12. 9, 12

13. 25, 40

14. 30, 70

15. 50, 75

16. 40, 100

17. 5, 8

18. 3, 10

19. 16, 28

20. 36, 42

21. 28, 98

22. 28, 70

23. 2, 3, 11

24. 3, 5, 7

25. 2, 5, 10

26. 3, 4, 8

27. 4, 6, 9

28. 4, 14, 35

29. 6, 10, 12

30. 10, 12, 20

31. 15, 45, 90

32. 30, 60, 120

33. 10, 20, 30, 40

34. 15, 25, 30, 40

For each set of numbers, **a.** find the LCM and **b.** state how many times each number divides into the LCM. See Examples 6 through 8.

35. 10, 15, 25

36. 14, 35, 49

37. 6, 24, 30

38. 15, 30, 105

39. 12, 18, 27

40. 10, 18, 90

41. 20, 56, 45

42. 40, 56, 196

For each equation, find the missing numerator that will make the fractions equivalent. See Examples 10 through 12.

43. $\frac{5}{8} = \frac{?}{24}$

44. $\frac{2}{5} = \frac{?}{25}$

45. $\frac{1}{16} = \frac{?}{64}$

46. $\frac{1}{9} = \frac{?}{45}$

47. $\frac{6}{7} = \frac{?}{49}$

48. $\frac{6}{5} = \frac{?}{45}$

49. $\frac{3}{4} = \frac{?}{20}$

50. $\frac{5}{8} = \frac{?}{16}$

51. $\frac{10}{11} = \frac{?}{44}$

52. $\frac{5}{21} = \frac{?}{42}$

53. $\frac{5}{8} = \frac{?}{96}$

54. $\frac{5}{12} = \frac{?}{108}$

55. $\frac{11}{12} = \frac{?}{48}$

56. $\frac{4}{7} = \frac{?}{35}$

57. $\frac{2}{3} = \frac{?}{48}$

58. $\frac{2}{5} = \frac{?}{65}$

59. $\frac{9}{10} = \frac{?}{90}$

60. $\frac{7}{10} = \frac{?}{80}$

61. $\frac{9}{10} = \frac{?}{100}$

62. $\frac{3}{10} = \frac{?}{100}$

63. $\frac{7}{6} = \frac{?}{36}$

64. $\frac{2}{3} = \frac{?}{27}$

65. $\frac{10}{3} = \frac{?}{33}$

66. $\frac{9}{7} = \frac{?}{84}$

Applications

Solve.

67. Three security guards meet at the front gate for coffee before they walk around inspecting buildings at a manufacturing plant. The guards take 15, 20, and 30 minutes, respectively, for the inspection trip.

a. If they start at the same time, in how many minutes will they meet again at the front gate for coffee?

b. How many trips will each guard have made?

68. Three trucks drivers have dinner together whenever all three are at the routing station at the same time. The first driver's route takes 6 days, the second driver's route takes 8 days, and the third driver's route takes 12 days.

a. How frequently do the three drivers have dinner together?

b. How frequently do the first two drivers meet?

69. Two astronauts miss connections at their first meeting in space.

a. If one astronaut circles the earth every 15 hours and the other every 18 hours, in how many hours will they meet again at the same place?

b. How many more orbits will each astronaut have to complete between missing their first meeting and making their second meeting?

70. Three neighbors mow their lawns at different intervals during the summer months. The first one mows every 5 days, the second every 7 days, and the third every 10 days.

a. How frequently do they mow their lawns on the same day?

b. How many times does each neighbor mow in between the times when they all mow together?

71. Four women work for the same book company selling textbooks. They leave the home office on the same day and take 8 days, 12 days, 14 days, and 15 days, respectively, to visit schools in their own sales regions.

a. In how many days will they all meet again at the home office?

b. How many sales trips will each have made in this time?

72. A fruit production company has three packaging facilities, each of which uses different-sized boxes as follows: 24 pieces/box, 36 pieces/box, and 45 pieces/box.

a. Assuming that the truck provides the same quantity of uniformly-sized pieces of fruit to all three packaging facilities, what is the minimum number of pieces of fruit that will be delivered so that no fruit will be left over?

b. How many boxes will each facility package?

73. Three swimmers decide to swim laps together, and they will quit when they reach the starting end of the pool together. The first swimmer can swim a lap in 35 seconds, the second will take 40 seconds, and the third takes 42 seconds.

a. How many seconds will it take before they quit?

b. How many laps will each swimmer swim in that interval?

74. Two analog clocks are sitting next to each other. The first clock keeps perfect time, where the minute hand takes 60 minutes to travel completely around the dial. The second clock runs fast and the minute hand makes one complete revolution in 55 minutes.

a. Assuming that both clocks are started so that the minute hands are at 12, how many minutes will it take until both minute hands return to 12 at the same time?

b. How many hours does this represent?

75. Two satellites are in polar orbit around the earth. One takes 8 hours to complete its orbit and the other 15 hours.

a. Assuming that the satellites start over the North Pole, how long will it take before they are both over the North Pole again?

b. How many orbits does each satellite make during this time?

76. Three cars are being driven around the same road circuit. Car 1 can drive it in 10 minutes, car 2 takes 20 minutes, and car 3 takes 40 minutes.

a. Assuming that all the cars start at the same time, how long will it take for them to reach the starting point together?

b. How long will it take for the first two cars to meet?

c. How long will it take for cars 1 and 3 to meet?

77. Four ships leave the port on the same day. They take 12, 15, 18, and 30 days, respectively, to sail their routes and reload cargo. How frequently do these four ships leave this port on the same day?

78. A small college offers some courses at intervals greater than one year. In one department of this college, four courses are each offered at different intervals as follows. One course is offered every 2 years, a second course is held every 3 years, a third course is given every 4 years, and a special elective is offered only once every 6 years. How long will it take for these courses to be offered during the same year after the previous time this happened?

79. A certain star has four planets, all moving in the same plane. The first one takes 4 years to go around the star, the second takes 6 years, the third 10 years, and the fourth takes 15 years. Assuming that they are initially aligned, how many years will it take until they are aligned again?

80. Four tour buses depart the terminal on the same day. The itinerary of the first bus takes 6 days, it takes 12 days for the second bus, it takes 14 days for the third bus, and it takes 21 days for the fourth bus. Assuming that another tour begins the same day that the bus returns to the terminal, how many days will it take until all four buses return to the terminal on the same day?

81. A box of 6 peanut clusters is sold for $3. How much would a box of 24 peanut clusters cost if the price per peanut cluster remains the same?

82. For fieldtrips, school policy requires that 2 adults are present for every 13 students. How many adults are needed on a fieldtrip for 52 students?

Writing & Thinking

83. Explain, in your own words, why each number in a set divides evenly into the LCM of that set of numbers.

84. Explain, in your own words, why the LCM of a set of numbers is greater than or equal to each number in the set.

85. Explain why simply multiplying two numbers together will not necessarily find the LCM of those numbers. Give an example of when it would find the LCM and an example when it would not.

86. There are several ways to find the least common multiple. Discuss the method you prefer and explain why.