

OR

$$\begin{aligned}
 \log_b \sqrt{x} + \log_b \sqrt[3]{x} &= \log_b x^{\frac{1}{2}} + \log_b x^{\frac{1}{3}} \\
 &= \log_b \left(x^{\frac{1}{2}} \cdot x^{\frac{1}{3}} \right) && \text{Product rule} \\
 &= \log_b x^{\left(\frac{1}{2} + \frac{1}{3} \right)} \\
 &= \log_b x^{\frac{5}{6}}
 \end{aligned}$$

Now work margin exercise 5.**Common Misunderstandings about Logarithms**

There is no logarithmic property for the logarithm of a sum or a difference.

$$\log_b (x + y) \quad \text{Cannot be simplified}$$

$$\log_b (x - y) \quad \text{Cannot be simplified}$$

Also,

$$\log_b (xy) \neq \log_b x \cdot \log_b y \quad \text{The log of a product does not equal the product of the logs.}$$

$$\log_b \frac{x}{y} \neq \frac{\log_b x}{\log_b y} \quad \text{The log of a quotient does not equal the quotient of the logs.}$$

CAUTION**Margin Exercise Answers**

1. a. 4 b. 1.6021 c. 1.7782 2. a. -0.4771 b. -2 c. 0.3802 3. a. 0.2386 b. 1.4314 c. 1.5563

4. a. $\log_b 3 + 2\log_b x$ b. $3\log_b x + \log_b y - \log_b z$ c. $-2\log_b m - 2\log_b n$ d. $\frac{1}{2}\log_b 2 + \frac{1}{2}\log_b a$

5. a. $\log_b \left(\frac{x^3}{y^4} \right)$ b. $\log_d \left(\frac{3}{4x} \right)$ c. $\log_a (y^2 - 4)$ d. $\log_b y^{\frac{3}{4}}$

17.5 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. Because logarithms are _____, their properties are similar to those of _____.
2. The logarithm of a product is equal to the _____ of the logarithms of the factors.
3. The logarithm of a quotient is equal to the _____ _____ the logarithm of the numerator and the logarithm of the denominator.
4. The logarithm of a number raised to a power is equal to the _____ of the exponent and the logarithm of the number.

17. $\log_6 \frac{2x}{y^3}$

18. $\log_3 \frac{xy}{4z}$

19. $\log_b \frac{x^2}{yz}$

20. $\log_3 \frac{xy^2}{z^2}$

21. $\log_5 (xy)^{-2}$

22. $\log_b (x^2y)^4$

23. $\log_6 \sqrt[3]{xy^2}$

24. $\log_5 \sqrt{2x^3y}$

25. $\log_3 \sqrt{\frac{xy}{z}}$

26. $\log_6 \sqrt[3]{\frac{x^2}{y}}$

27. $\log_5 21x^2y^{\frac{2}{3}}$

28. $\log_b 15x^{\frac{1}{2}}y^{\frac{1}{3}}$

29. $\log_6 \frac{x}{\sqrt{x^3y^5}}$

30. $\log_3 \frac{1}{\sqrt{x^4y}}$

31. $\log_b \left(\frac{x^3y^2}{z} \right)^{-3}$

32. $\log_4 \left(\frac{x^{\frac{1}{2}}y}{z^2} \right)^{-2}$

Use the properties of logarithms to write each expression as a single logarithm of a single expression. See Example 5.

33. $2 \log_b 3 + \log_b x - \log_b 5$

34. $\frac{1}{2} \log_b 25 + \log_b 3 - \log_b x$

35. $\log_2 7 + \log_2 9 + 2 \log_2 x$

36. $\log_5 4 + \log_5 6 + \log_5 y$

37. $2 \log_b x + \log_b y$

38. $\log_2 x + 3 \log_2 y$

39. $3 \log_5 y - \frac{1}{2} \log_5 x$

40. $3 \log_{10} x - 2 \log_{10} y$

41. $\frac{1}{2} (\log_5 x - \log_5 y)$

42. $\frac{1}{3} (\log_{10} x - 2 \log_{10} y)$

43. $\log_2 x - \log_2 y + \log_2 z$

44. $\log_b x - 2 \log_b y - 2 \log_b z$

45. $\log_b x + 2 \log_b y - \frac{1}{2} \log_b z$

46. $-\frac{2}{3} \log_2 x - \frac{1}{3} \log_2 y + \frac{2}{3} \log_2 z$

47. $2 \log_5 x + \log_5 (2x + 1)$

48. $\log_b (3x + 1) + 2 \log_b x$

49. $\log_2 (x - 1) + \log_2 (x + 3)$

50. $\log_{10} (x + 3) + \log_{10} (x - 3)$

51. $\log_b (x^2 - 2x - 3) - \log_b (x - 3)$

52. $\log_2 (x - 4) - \log_2 (x^2 - 2x - 8)$

53. $\log_{10} (x + 6) - \log_{10} (2x^2 + 9x - 18)$

54. $\log_5 (3x^2 + 5x - 2) - \log_5 (3x - 1)$

Writing & Thinking

55. Prove the quotient rule for logarithms: For $b > 0$, $b \neq 1$, and $x, y > 0$,

$$\log_b \frac{x}{y} = \log_b x - \log_b y.$$

56. Prove the following property of logarithms: For $b > 0$, $b \neq 1$, and $x > 0$, $\log_b b^x = x$.