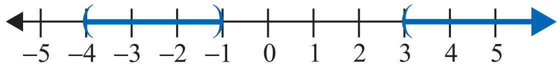
Thus, the solution set is the union of two intervals: $(-4, -1) \cup (3, \infty)$.



Now work margin exercise 11.

Margin Exercise Answers

1. $(-5, 2)$ **2.** $(-\infty, -4] \cup \left[\frac{1}{2}, \infty\right)$ **3.** $(-1, 0) \cup (7, \infty)$ **4.** $\left(\frac{1-\sqrt{21}}{2}, \frac{1+\sqrt{21}}{2}\right)$

5. $\varnothing$ (No Solution) **6.** $(-\infty, -1) \cup (5, \infty)$ **7.** $(-6, -4]$ **8.** $\left[\frac{7}{3}, 5\right)$

9. $(-\infty, -1) \cup (4, \infty)$ **10.** $(-1.8508, 1.3508)$ **11.** $(-\infty, -4) \cup (1, 2)$

16.8 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. Quadratic and rational inequalities can be solved by factoring or using the quadratic formula and then analyzing the ______ of the corresponding functions.

2. The technique of factoring to solve inequalities is based on the idea that for values of x on either side of a number a, the ______ for an expression of the form $(x - a)$ ______.

3. When solving a polynomial inequality algebraically, mark the points where each factor is 0 on the number line. These are the interval ______.

4. After marking the points on the number line, test one point from each interval to determine the ______ of the polynomial expression for all points in that interval.

5. On a number line, use a/an ______ for an endpoint that is included and a/an ______ for an endpoint that is not included.

6. When solving rational inequalities, mark the points on a number line where each factor is 0 or causes the ______ to be 0.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. When solving a polynomial inequality algebraically, the goal is to get the constants on one side of the inequality and to factor the polynomial on the other side.

8. Test points are used to determine which intervals on the number line satisfy the original inequality.

9. The solution of a polynomial inequality is a single interval.

10. If an endpoint causes the denominator of a rational inequality to be 0, it should be marked with a parenthesis.

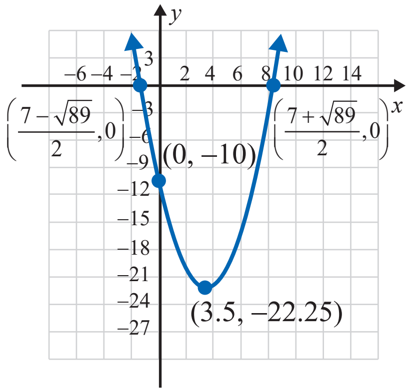
Practice

Solve the quadratic (and higher degree) inequalities algebraically. Write the answers in interval notation, and then graph each solution set on a number line. (**Note:** You may need to use the quadratic formula to find endpoints of intervals.) See Examples 1 through 5.

1. $(x-6)(x+2)<0$
2. $(x+4)(x-2)>0$
3. $(3x-2)(x-5)>0$
4. $(4x+1)(x+1)\le 0$
5. $(x+7)(2x-5)\ge 0$
6. $(x-3)(5x-3)\le 0$
7. $(3x+1)(x+2)\le 0$
8. $(x-4)(3x-8)>0$
9. $x(3x+4)(x-5)<0$
10. $(x-1)(x+4)(2x+5)<0$
11. $x^2+4x+4\le 0$
12. $5x^2+4x-12>0$
13. $2x^2>x+15$
14. $6x^2+x>2$
15. $8x^2<10x+3$
16. $2x^2<x+10$
17. $2x^2-5x+2\ge 0$
18. $15y^2-21y-18<0$
19. $6y^2+7y<-2$
20. $3x^2+3\ge 10x$
21. $4z^2-20z+25>0$
22. $15x^2-11x-14\le 0$
23. $8x^2+6x\le 35$
24. $7x<6x^2+x^3$
25. $x^3>2x^2+3x$
26. $x^3<6x^2-9x$
27. $x^3>5x^2-4x$
28. $4x^2\le x^3+3x$
29. $(x+2)(x-2)>3x$
30. $(x+4)(x-1)<2x+2$
31. $x^4-5x^2+4>0$
32. $x^4-25x^2+144<0$
33. $y^4-13y^2+36\le 0$
34. $y^4-13y^2-48\ge 0$
35. $(x+1)^2-9\ge 0$
36. $(3x-1)^2-16<0$
37. $(2x-3)(3x+2)-(3x+2)<0$
38. $2(x-1)(x-3)>(x-1)(x-6)$
39. $x^2+2x-4>0$
40. $x^2-8x+14<0$
41. $x^2+6x+7\ge 0$
42. $2x^2+4x-3<0$
43. $3x^2+5x+1<0$
44. $3x^2+8x+5\ge 0$
45. $2x^3\le 7x^2+4x$
46. $2x^2>9x-8$
47. $x^2-2x+2>0$
48. $x^2+3x+3<0$
49. $2x-1>3x^2$
50. $6x-10<x^2$

The graph of a quadratic function is given. Use the information in the graph to solve the related equations and inequalities in parts a. through c.

51. $y = x^2 - 7x - 10$

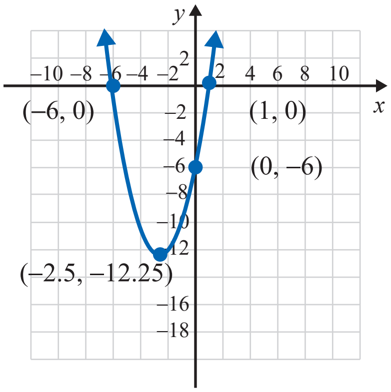


a. $x^2 - 7x - 10 = 0$

b. $x^2 - 7x - 10 > 0$

c. $x^2 - 7x - 10 < 0$

52. $y = x^2 + 5x - 6$

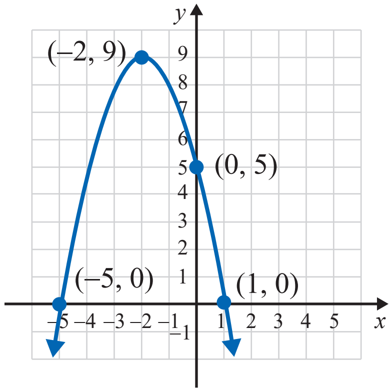


a. $x^2 + 5x - 6 = 0$

b. $x^2 + 5x - 6 > 0$

c. $x^2 + 5x - 6 < 0$

53. $y = -x^2 - 4x + 5$

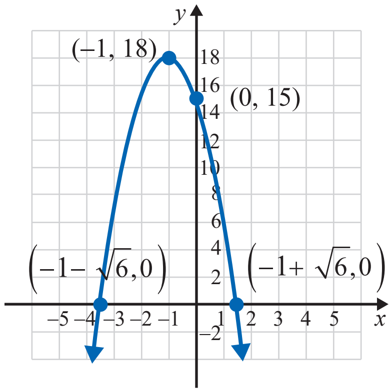


a. $-x^2 - 4x + 5 = 0$

b. $-x^2 - 4x + 5 > 0$

c. $-x^2 - 4x + 5 < 0$

54. $y = -3x^2 - 6x + 15$



a. $-3x^2 - 6x + 15 = 0$

b. $-3x^2 - 6x + 15 > 0$

c. $-3x^2 - 6x + 15 < 0$

Solve the rational inequalities algebraically. Write the answers in interval notation, and then graph each solution set on a number line. See Examples 6 through 8.

55. $\frac{x+4}{2x} \geq 0$

56. $\frac{x}{x-4} \geq 0$

57. $\frac{x+6}{x^2} < 0$

58. $\frac{3x^2}{x+1} < 0$

59. $\frac{x+3}{x+9} > 0$

60. $\frac{2x+3}{x-4} < 0$

61. $\frac{3x-6}{2x-5} < 0$

62. $\frac{4-3x}{2x+4} \leq 0$

63. $\frac{x+5}{x-7} \geq 1$

64. $\frac{2x+3}{x-1} > 2$

65. $\dfrac{2x+5}{x-4} \le -3$

66. $\dfrac{3x+2}{4x-1} < 3$

67. $\dfrac{5-2x}{3x+4} < -1$

68. $\dfrac{8-x}{x+5} < -4$

69. $\dfrac{x(x+4)}{x-3} \le 0$

70. $\dfrac{(x+3)(x-2)}{x+1} > 0$

71. $\dfrac{x-5}{x(x+2)} \ge 0$

72. $\dfrac{-(x-3)^2}{(x-1)(x-4)} < 0$

Use a graphing calculator to solve the inequalities. Write the answers in interval notation, and then graph each solution set on a number line. (Estimate endpoints, when necessary, to 4 decimal places.)

73. $x^2 > 10$

74. $20 \ge x^2$

75. $x^2 - 2.5x + 6.25 < 0$

76. $x^2 + 2x \ge -1$

77. $x^3 - 9x < 0$

78. $x^3 - 4x^2 + 4x \le 0$

79. $2x^3 - 5x + 4 \ge 0$

80. $x^3 - 4x^2 + 3 < 0$

81. $-x^4 + 6x^2 - 3 > 0$

82. $x^4 - 2x^3 - x^2 - 1 < 0$

Applications

Solve.

83. A high school student is selling T-shirts to raise money for the band. She realizes that the number of shirts she sells each week can be modeled by $f(x) = -x^2 + 12x - 17$, where x is the amount she charges per shirt. Solve the inequality $-x^2 + 12x - 17 \ge 10$ to find the range she can charge per shirt and sell at least 10 shirts in a week.

84. Maria tracked the nighttime temperatures for a week and noticed that the temperature, in Celsius, could be modeled by $f(x) = \frac{1}{2}x^2 - 4x + 6$, where x is the number of hours after midnight. Maria has plants that might die if they are left out when the temperature drops below freezing (0 degrees Celsius). Solve the inequality $\frac{1}{2}x^2 - 4x + 6 < 0$ to find timeframe in which her plants will be in danger.

Writing & Thinking

85. Use a graphing calculator to graph the rational function $y = \dfrac{x^2 + 3x - 4}{x}$.

a. Use the graph to find the solution set for $y > 0$.

b. Use the graph to find the solution set for $y < 0$.

c. Explain the effect of $x = 0$ on the graph and why $x = 0$ is not included in either parts a. or b.

86. In your own words, explain why (as in Example 5), when the quadratic formula gives nonreal values, the quadratic polynomial is either always positive or always negative.