The maximum area occurs at the point where $x = -\frac{b}{2a} = -\frac{240}{-4} = 60$.

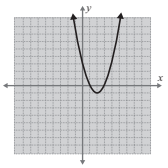
Two sides of the rectangle are 60 feet and the third side is $240 - 2(60) = 120$ feet.

The maximum area possible is $60(120) = 7200$ square feet.

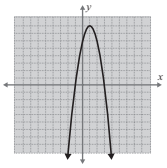
Now work margin exercise 6.

Margin Exercise Answers

1. $x = 2$; vertex: $(2, -1)$; x-int: $(1, 0), (3, 0)$; y-int: $(0, 3)$;



2. $x = 1$; vertex: $(1, 8)$; x-int: $(3, 0), (-1, 0)$; y-int: $(0, 6)$;



3. $x = -3$; vertex: $(-3, 12)$; x-int: $\left(-3 + 2\sqrt{3}, 0\right), \left(-3 - 2\sqrt{3}, 0\right)$; y-int: $(0, 3)$

4. $x = 1$; vertex: $(1, 3)$; x-int: none; y-int: $(0, 4)$; **5.** It will take the ball 1.5 sec to reach its maximum height of 36 ft. **6.** Two sides of the lot are 90 yards and the third side is 180 yards for a maximum area of 16,200 square yards.

16.7 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. Quadratic functions of the form $y = a(x - h)^2 + k$ have a/an _____ at (h, k).

2. The points where a parabola crosses the x-axis, if any, are the x-intercepts. These points are also called the _____ of the function.

3. If the solutions of a quadratic function are nonreal complex numbers, then the graph does not cross the _____.

4. If $a > 0$, then the parabola opens _____ and (h, k) is the _____ point and the y-value k is called the _____ value of the function.

5. If $a < 0$, then the parabola opens _____ and (h, k) is the _____ point and the y-value k is called the _____ value of the function.

6. The x-intercepts, or _____, of a function can be found by substituting _____ for y and solving the resulting quadratic equation.

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True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. Quadratic functions of the form $y = a(x - h)^2 + k$ have a line of symmetry at $x = \frac{b}{2a}$.

8. The vertex of a vertical parabola is the lowest point on the parabola.

9. The maximum or minimum value of a quadratic function written in general form can be found by letting $x = -\frac{b}{2a}$ and solving for y.

10. When the solutions to a quadratic function are nonreal, the entire graph lies either completely above or below the x-axis.

Practice

Rewrite each quadratic function in the form $y = a(x - h)^2 + k$. Find the line of symmetry, the vertex, the x-intercepts, and the y-intercept. Graph the function. See Examples 1 and 2.

1. $y = 2x^2 - 4x + 2$

2. $y = -3x^2 + 12x - 12$

3. $y = x^2 - 2x - 3$

4. $y = x^2 - 4x + 5$

5. $y = x^2 + 6x + 5$

6. $y = x^2 - 8x + 12$

7. $y = 2x^2 - 8x + 5$

8. $y = 2x^2 - 12x + 16$

9. $y = -3x^2 - 12x - 9$

10. $y = 3x^2 - 6x - 1$

11. $y = 5x^2 - 10x + 8$

12. $y = -4x^2 + 16x - 11$

13. $y = -x^2 - 5x - 2$

14. $y = x^2 + 3x - 1$

15. $y = 2x^2 + 7x + 5$

16. $y = 2x^2 + x - 3$

For each quadratic function use the formula $x = -\frac{b}{2a}$ to find the line of symmetry and the vertex. Then find the x-intercepts and the y-intercept. Graph the function. See Examples 3 and 4.

17. $y = -3x^2 + 6x - 3$

18. $y = x^2 - 2x - 8$

19. $y = x^2 - 4x + 3$

20. $y = x^2 + 6x + 5$

21. $y = -2x^2 + 8x - 9$

22. $y = 5x^2 + 10x + 7$

23. $y = x^2 + 2x + 1$

24. $y = x^2 + 8x + 7$

25. $y = -x^2 - 2x - 2$

26. $y = 2x^2 + 4x - 6$

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Graph the two given functions and answer the following questions:

a. Are the graphs the same?
b. Do the functions have the same zeros?
c. Briefly, discuss your interpretation of the results in parts a. and b.

27. $\begin{cases} y = x^2 - 3x - 10 \\ y = -x^2 + 3x + 10 \end{cases}$

28. $\begin{cases} y = x^2 - 5x + 6 \\ y = -x^2 + 5x - 6 \end{cases}$

29. $\begin{cases} y = 2x^2 - 5x - 3 \\ y = -2x^2 + 5x + 3 \end{cases}$

30. $\begin{cases} y = -4x^2 - 15x + 4 \\ y = 4x^2 + 15x - 4 \end{cases}$

Use the CALC features of the calculator to find the zeros of the function. (**Hint:** The `zero` item on the CALC menu will locate the zeros of the function.) Round answers to nearest ten-thousandth.

31. $y = x^2 - 2x - 2$

32. $y = 3x^2 + x - 1$

33. $y = -2x^2 + 2x + 5$

34. $y = -x^2 - 2x + 7$

35. $y = x^2 + 3x + 3$

36. $y = -4x^2 - x - 6$

Use a graphing calculator to graph each function by pressing [Y=] and entering the function. Find the coordinates of the maximum as follows. Round answers to nearest ten-thousandth.

Step 1: Press CALC ([2nd] [TRACE]).

Step 2: Press or choose `maximum`.

Step 3: Follow the directions for moving the cursor to `Left Bound`?, `Right Bound`?, and `Guess`?. (Press [ENTER] each time.)

37. $y = 4x - x^2$

38. $y = 1 - 2x - x^2$

39. $y = -8 + 4x - x^2$

40. $y = 3 - 2x - x^2$

Use a graphing calculator to graph each function by pressing [Y=] and entering the function. Find the coordinates of the minimum as follows. Round answers to nearest ten-thousandth.

Step 1: Press CALC ([2nd] [TRACE]).

Step 2: Press or choose `minimum`.

Step 3: Follow the directions for moving the cursor to `Left Bound?`, `Right Bound?`, and `Guess?`. (Press [ENTER] each time.)

41. $y = x^2 - 8x + 15$

42. $y = x^2 + 10x + 22$

43. $y = 2x^2 + 4x + 3$

44. $y = 3x^2 - 6x + 5$

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Applications

Use the function $h = -16t^2 + v_0 t + h_0$, where h is the height of the object after time t, v_0 is the initial velocity, and h_0 is the initial height. See Example 5.

45. A ball is thrown vertically upward from the ground with an initial velocity of 112 ft/s.

a. When will the ball reach its maximum height?

b. What will be the maximum height?

46. A water rocket is launched from the ground and has an initial velocity of 104 ft/s.

a. When will the rocket reach its maximum height?

b. What will be the maximum height?

47. A stone is projected vertically upward from a platform that is 20 feet high at a rate of 160 feet per second.

a. When will the stone reach its maximum height?

b. What will be the maximum height?

48. A cannonball is projected vertically upward from a platform that is 32 feet high at a rate of 128 feet per second.

a. When will the cannonball reach its maximum height?

b. What will be the maximum height?

Solve.

49. A retailer sells fitness trackers. He estimates that by selling them for x dollars each, he will be able to sell $100 - x$ fitness trackers each month.

a. What price will yield maximum revenue?

b. What will be the maximum revenue?

50. Mrs. Richey can sell 72 picture frames each month if she charges \$24 each. She estimates that for each \$1 increase in price, she will sell 2 fewer frames.

a. Find the price that will yield maximum revenue.

b. What will be the maximum revenue?

51. A store owner estimates that by charging x dollars each for a certain lamp, he can sell $40 - x$ lamps each week. What price will give him maximum sales revenue?

52. The band Pumpkin Riot estimates that by selling T-shirts for x dollars each, they can sell $250 - 10x$ T-shirts per show they play. Determine the price per T-shirt that will give the band maximum sales revenue.

53. A nature reserve plans to fence off an area of land to restore balance to the native plant and wildlife populations. The fence will be on three sides and form a rectangle, with a river along the fourth side. The planning committee determines they have enough funds to install 600 yards of fencing.

a. What dimensions should the fence have to enclose the maximum area?

b. What is the maximum area that can be enclosed?

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54. A contractor is to build a six-foot-high brick wall to enclose a rectangular garden. The wall will be on three sides of the rectangle while the fourth side is a building. The owner wants to enclose the maximum area but only wants to pay for 150 feet of wall. What dimensions should the contractor make the garden?

Writing & Thinking

55. Discuss the following features of the general quadratic function $y = ax^2 + bx + c$.

a. What type of curve is its graph?

b. What is the value of x at its vertex?

c. What is the equation of the line of symmetry?

d. Does the graph always cross the x-axis? Explain.

56. Discuss the discriminant of the general quadratic equation $ax^2 + bx + c = 0$ and how the value of the discriminant is related to the graph of the corresponding quadratic function $y = ax^2 + bx + c$.

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