

Completion Example Answers

$$\begin{aligned}
 6. \quad 2x^2 - 12x &= \underline{-2} \\
 x^2 - 6x &= \underline{-1} \\
 x^2 - 6x + 9 &= \underline{-1} + \underline{9} \\
 (x - 3)^2 &= \underline{8} \\
 x - 3 &= \pm\sqrt{8} \\
 x &= 3 \pm 2\sqrt{2}
 \end{aligned}$$

Margin Exercise Answers

$$\begin{aligned}
 1. \text{ a. } y^2 - 14y + 49 &= (y - 7)^2 & \text{ b. } x^2 + 9x + \frac{81}{4} &= \left(x + \frac{9}{2}\right)^2 & 2. \quad x &= 5 \pm 2\sqrt{14} & 3. \quad x &= -1 \pm \sqrt{7} \\
 4. \quad x &= \frac{-1 \pm \sqrt{19}}{2} & 5. \quad x &= -2 \pm i\sqrt{7} & 6. \quad x &= 1 \pm \sqrt{6} & 7. \quad y^2 - 8y + 25 &= 0 & 8. \quad x^2 - 4x - 14 &= 0 \\
 9. \quad x^2 - 2x + 6 &= 0
 \end{aligned}$$

16.2 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

- Completing the square is the process of adding terms to binomials so that the result will be a perfect square _____.
- To solve a quadratic equation by completing the square, arrange terms with _____ on one side of the equation and _____ on the other.
- When solving by completing the square, the quadratic equation should have a leading coefficient of _____.
- After finding the coefficient that completes the square of the polynomial, _____ this constant to both sides of the equation.
- When completing the square, the constant term is the _____ of $\frac{1}{2}$ of the coefficient of x .

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

- To get a leading coefficient of 1, multiply both sides of the equation by the reciprocal of the leading coefficient.
- When solving a quadratic equation, there is either no solution or two solutions.
- It's possible for the roots of a quadratic equation to be nonreal numbers.
- The last step of solving a quadratic equation by completing the square is to use the square root property.

Practice

Add the correct constant to complete the square; then factor the trinomial as indicated. See Example 1.

1. $x^2 - 12x + \underline{\hspace{1cm}} = (\underline{\hspace{1cm}})^2$

2. $y^2 + 14y + \underline{\hspace{1cm}} = (\underline{\hspace{1cm}})^2$

3. $x^2 + 6x + \underline{\hspace{1cm}} = (\underline{\hspace{1cm}})^2$

4. $x^2 + 8x + \underline{\hspace{1cm}} = (\underline{\hspace{1cm}})^2$

5. $x^2 - 5x + \underline{\hspace{1cm}} = (\underline{\hspace{1cm}})^2$

6. $x^2 + 7x + \underline{\hspace{1cm}} = (\underline{\hspace{1cm}})^2$

7. $y^2 + y + \underline{\hspace{1cm}} = (\underline{\hspace{1cm}})^2$

8. $x^2 + \frac{1}{2}x + \underline{\hspace{1cm}} = (\underline{\hspace{1cm}})^2$

9. $x^2 + \frac{1}{3}x + \underline{\hspace{1cm}} = (\underline{\hspace{1cm}})^2$

10. $y^2 + \frac{3}{4}y + \underline{\hspace{1cm}} = (\underline{\hspace{1cm}})^2$

11. $2x^2 + 4x + \underline{\hspace{1cm}} = 2(\underline{\hspace{1cm}})^2$

12. $3x^2 + 18x + \underline{\hspace{1cm}} = 3(\underline{\hspace{1cm}})^2$

Solve the quadratic equations by completing the square. See Examples 2 through 6.

13. $x^2 + 4x - 5 = 0$

31. $z^2 + 3z - 5 = 0$

14. $x^2 + 6x - 7 = 0$

32. $x^2 - 5x + 5 = 0$

15. $y^2 + 2y = 5$

33. $x^2 + x + 2 = 0$

16. $x^2 + 3 = 8x$

34. $y^2 + 3y + 3 = 0$

17. $x^2 + 3 = 10x$

35. $x^2 + 5x + 2 = 0$

18. $z^2 + 4z = 2$

36. $4x^2 + 7x + 2 = 0$

19. $x^2 - 4x - 45 = 0$

37. $3x^2 - 10x + 5 = 0$

20. $x^2 - 10x + 21 = 0$

38. $3y^2 + 5y - 3 = 0$

21. $x^2 - 3x - 40 = 0$

39. $3x^2 + 6x + 18 = 0$

22. $x^2 + x - 42 = 0$

40. $4x^2 + 8x + 16 = 0$

23. $3x^2 + x - 4 = 0$

41. $2x - 3 = 4x^2$

24. $2x^2 + x - 6 = 0$

42. $2x + 2 = -6x^2$

25. $x^2 - 6x + 10 = 0$

43. $5y^2 + 15y + 25 = 0$

26. $x^2 - 2x + 5 = 0$

44. $4x^2 + 20x + 32 = 0$

27. $x^2 + 11 = 12x$

45. $3y^2 = 4 - y$

28. $x^2 = 6 - x$

46. $2x^2 + 4 = -9x$

29. $y^2 = 10y - 4$

47. $2x^2 - 8x + 4 = 0$

30. $x^2 = 3 - 4x$

48. $3x^2 - 18x + 12 = 0$

Write a quadratic equation with integer coefficients that has the given roots.
See Examples 7 through 9.

49. $x = \sqrt{7}, x = -\sqrt{7}$

50. $x = \sqrt{6}, x = -\sqrt{6}$

51. $x = 1 + \sqrt{3}, x = 1 - \sqrt{3}$

52. $z = 3 + \sqrt{2}, z = 3 - \sqrt{2}$

53. $y = -2 + 2\sqrt{5}, y = -2 - 2\sqrt{5}$

54. $x = 1 + 2\sqrt{3}, x = 1 - 2\sqrt{3}$

55. $x = 4i, x = -4i$

56. $x = 7i, x = -7i$

57. $y = i\sqrt{6}, y = -i\sqrt{6}$

58. $y = i\sqrt{5}, y = -i\sqrt{5}$

59. $x = 2 + i, x = 2 - i$

60. $x = -3 + 2i, x = -3 - 2i$

61. $x = 1 + i\sqrt{2}, x = 1 - i\sqrt{2}$

62. $x = 2 + i\sqrt{3}, x = 2 - i\sqrt{3}$

63. $x = -5 + 2i\sqrt{6}, x = -5 - 2i\sqrt{6}$

64. $y = 4 + 3i\sqrt{2}, y = 4 - 3i\sqrt{2}$

Applications

Solve.

65. A local frame shop determines that the revenue function for their custom framing service is $R(p) = 360p - 4p^2$, where p is the base price in dollars for each custom framing job.
- Set the function equal to 0 and solve for p using the method of completing the square.
 - What do the solutions from part a. mean?
66. The height of a golf ball that is hit from the ground at a speed of 128 feet per second can be modeled with the expression $h(t) = -16t^2 + 128t$, where t is the time in seconds after the ball is hit.
- Set the function equal to 0 and solve for t using the method of completing the square.
 - What do the solutions from part a. mean?

Writing & Thinking

67. Explain, in your own words, the steps involved in the process of solving a quadratic equation by completing the square.