

Example 8 The Pythagorean Theorem

The diagonal of a rectangle is twice the width. The length of the rectangle is 6 feet. Find the width of the rectangle.

Solution

Let x = width of the rectangle
and $2x$ = length of the diagonal.

Then, using the Pythagorean Theorem, we have

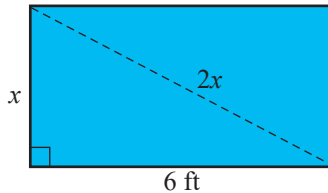
$$(2x)^2 = x^2 + 6^2$$

$$4x^2 = x^2 + 36$$

$$3x^2 = 36$$

$$x^2 = 12$$

$$x = \sqrt{12} = \sqrt{4} \cdot \sqrt{3} = 2\sqrt{3} (\approx 3.46).$$



The width of the rectangle is $2\sqrt{3}$ feet or approximately 3.46 feet. The negative solution to the quadratic equation is not considered because length is not negative.

Now work margin exercise 8.**Completion Example Answers**

6. 20 ; $\pm 2\sqrt{5}$; $-2 \pm 2\sqrt{5}$

Margin Exercise Answers

1. $x = 2, 8$ 2. $x = 3$ 3. $x = -5i, 5i$ 4. $x = -6 \pm 2\sqrt{3}$ 5. $\pm 4i$ 6. $x = 4 \pm 2\sqrt{3}$

7. 10 feet 8. $\frac{5\sqrt{2}}{2}$ inches (≈ 3.54 inches)

16.1 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

- In a right triangle, the square of the length of the _____ is equal to the sum of the _____ of the lengths of the two legs.
- The two equations $x = \sqrt{c}$ or $x = -\sqrt{c}$ can be written as _____.
- When using the square root method, the polynomial is not set to zero. Instead, the squared expression is set equal to a/an _____ real number.
- When using the square root method, you take the square root of _____ side(s).

- The diagonal of a rectangle is three times the width. The length of the rectangle is 10 inches. Find the width of the rectangle.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

5. Using the square root method on an equation of the form $x^2 = c$, where c is a nonnegative number, will always result in two distinct solutions.
6. Quadratic equations that are not easily solved using factoring might be solved by the square root method.
7. The square of a real number can be negative.

Practice

Solve the following quadratic equations by factoring. See Examples 1 through 3.

- | | |
|--------------------------|---|
| 1. $x^2 = 11x$ | 6. $5x^2 + 17x = -6$ |
| 2. $x^2 - 10x + 16 = 0$ | 7. $(x + 3)(x - 1) = 4x$ |
| 3. $x^2 = -15x - 36$ | 8. $(x - 7)(x - 2) = 6$ |
| 4. $2x^2 + 36x + 34 = 0$ | 9. $(2x - 3)(2x + 1) = 3x - 6$ |
| 5. $9x^2 + 6x - 15 = 0$ | 10. $(x - 2)(5x + 4) = 3x^2 - 15x - 12$ |

Solve the following quadratic equations by using the square root method. Write each radical in simplest form. See Examples 4 through 6.

- | | |
|--------------------|---------------------------------|
| 11. $x^2 = 121$ | 26. $4x^2 = 25$ |
| 12. $x^2 = 81$ | 27. $(x - 1)^2 = 4$ |
| 13. $3x^2 = 108$ | 28. $(x + 3)^2 = 9$ |
| 14. $5x^2 = 245$ | 29. $(x + 2)^2 = -25$ |
| 15. $x^2 = 35$ | 30. $(x - 5)^2 = 36$ |
| 16. $x^2 = 42$ | 31. $(x - 3)^2 = -4$ |
| 17. $x^2 + 25 = 0$ | 32. $(x + 8)^2 = -9$ |
| 18. $x^2 + 81 = 0$ | 33. $(x + 1)^2 = \frac{1}{4}$ |
| 19. $x^2 - 62 = 0$ | 34. $(x - 9)^2 = -\frac{9}{25}$ |
| 20. $x^2 - 75 = 0$ | 35. $(x - 2)^2 = \frac{1}{16}$ |
| 21. $x^2 - 45 = 0$ | 36. $(x - 3)^2 = \frac{4}{9}$ |
| 22. $x^2 - 98 = 0$ | 37. $(x + 2)^2 = -7$ |
| 23. $3x^2 = 54$ | 38. $(x + 8)^2 = 75$ |
| 24. $5x^2 = 60$ | |
| 25. $9x^2 = 4$ | |

39. $(5x-2)^2 = 63$

44. $3(x+11)^2 = 60$

40. $(4x-3)^2 = 125$

45. $3(x-5)^2 + 5 = -25$

41. $(3x+4)^2 + 3 = 30$

46. $2(x-6)^2 - 11 = 25$

42. $(2x+1)^2 + 12 = 60$

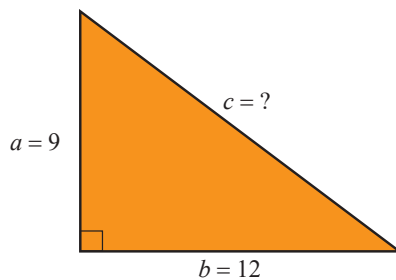
47. $-3(x^2+15) - 7 = 20$

43. $2(x-7)^2 = 24$

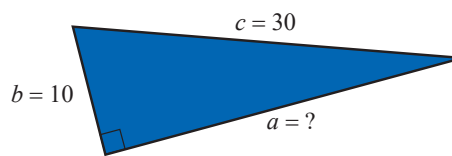
48. $-4(x^2+12)+1 = 25$

The lengths of two sides are given for each of the following right triangles. Determine the length of the missing side. See Examples 7 and 8.

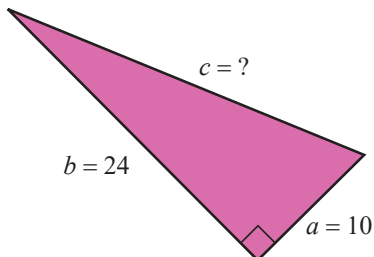
49. $a = 9, b = 12, c = ?$



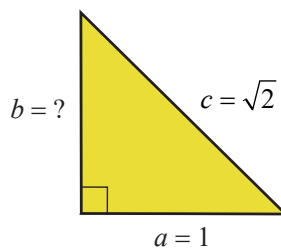
52. $b = 10, c = 30, a = ?$



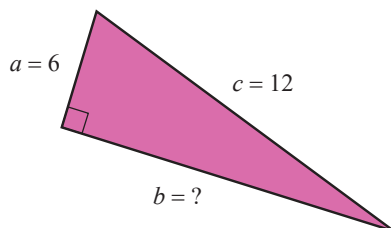
50. $a = 10, b = 24, c = ?$



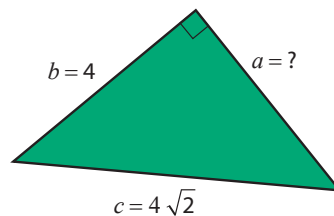
53. $a = 1, c = \sqrt{2}, b = ?$




51. $a = 6, c = 12, b = ?$



54. $b = 4, c = 4\sqrt{2}, a = ?$



 Use a calculator to solve the following quadratic equations. Round your answers to the nearest hundredth.

55. $x^2 = 647$

59. $6x^2 = 17.32$

56. $x^2 = 378$

60. $15x^2 = 229.63$

57. $19x^2 = 523$

61. $2.1x^2 = 35.82$

58. $14x^2 = 795$

62. $4.7x^2 = 118.34$

 Use a calculator to find the two values of each expression accurate to the nearest ten-thousandth.

63. $2 \pm \sqrt{5}$




64. $-4 \pm \sqrt{89}$


65. $\frac{2 \pm \sqrt{7}}{2}$

66. $\frac{5 \pm \sqrt{13}}{10}$

Applications

Solve.

67. The hypotenuse of a right triangle is twice the length of one of the legs. The length of the other leg is $4\sqrt{3}$ feet. Find the length of the leg and the hypotenuse.
68. One leg of a right triangle is three times the length of the other. The length of the hypotenuse is 20 cm. Find the lengths of the legs.
69. The two legs of a right triangle are the same length. The hypotenuse is 6 cm long. Find the length of the legs.
70. The two legs of a right triangle are the same length. The hypotenuse is $4\sqrt{2}$ m long. Find the length of the legs.
71.  A 38-foot ladder is leaning against a building that is 34 feet high. The top of the ladder extends 3 feet beyond the point where it touches the building.
- How far is the base of the ladder from the base of the building? Write your answer as a decimal number rounded to the nearest tenth.
 - According to OSHA guidelines, the distance from the base of the ladder to the building should be $\frac{1}{4}$ the distance from the ground to the point of contact. Does the distance of the base of the ladder to the building meet the safety guidelines? Explain your answer.
72.  The top of a telephone pole is 40 feet above the ground and a guy wire is to be stretched from the top of the pole to a point on the ground 10 feet from the base of the pole. For proper installation, the wire needs to be 7 feet longer than the distance from the top of the pole to the point on the ground. How long should the wire be?
73.  The library is located “around the corner” from the bank. If the library is 100 yards from the street corner and the bank is 75 yards around the corner on another street, what is the distance between the library and the bank “as the crow flies?”
74. A ball is dropped from the top of a building that is known to be 144 feet high. The formula for finding the height of the ball at any time is $h = 144 - 16t^2$ where t is measured in seconds. How many seconds will it take for the ball to hit the ground?
75. A ball is dropped from the top of a building that is 784 feet high. The height of the ball above ground level is given by the polynomial function $h(t) = -16t^2 + 784$ where t is measured in seconds.
- How high is the ball after 3 seconds? 5 seconds?
 - How far has the ball traveled in 3 seconds? 5 seconds?
 - When will the ball hit the ground? Explain your reasoning in terms of factors.

76. A tennis ball is dropped from a building. The position of the ball after t seconds is given by the polynomial function $s(t) = -4.9t^2 + 490$, where s is the height in meters of the ball.
- Find $s(0)$ What does this value represent in the context of this problem?
 - How high is the tennis ball 2 seconds after it has been dropped?
 - How long before the tennis ball hits the ground?
77.  An oil spill from a ruptured pipeline is circular in shape and covers an area of about 8 square miles. About how many miles long (to the nearest tenth of a mile) is the diameter of the oil spill? (Use $\pi = 3.14$.)
78. A financial consultant is asked for advice about finances and savings plans. When a client invests money, they need to know which interest rate will meet their financial goals based on the amount invested. The financial consultant can use the formula $A = P(r + 1)^n$ to find the future amount A , after n years, of an investment with a starting principal P invested at an interest rate of r .
- A client has \$3000 to invest and would like to earn \$300 on his investment after 2 years. At what interest rate will the client need to invest his money? Round to the nearest hundredth of a percent.
 - Another client has \$5000 to invest and would like to earn \$750 on her investment after 2 years. At what interest rate will the client need to invest her money? Round to the nearest hundredth of a percent.