

3. Simplify each power of i .

a. i^{37}

b. i^{14}

c. i^{-8}

Example 3 Simplifying Powers of i

Simplify each power of i .

a. $i^{45} = i^{44} \cdot i = (i^4)^{11} \cdot i = 1^{11} \cdot i = i$ $i = 0 + i$ in standard form.

b. $i^{58} = i^{56} \cdot i^2 = (i^4)^{14} \cdot i^2 = 1^{14} \cdot (-1) = -1$ $1 = -1 + 0i$ in standard form.

c. $i^{-7} = \frac{1}{i^7} = \frac{1}{i^7} \cdot \frac{i}{i} = \frac{i}{i^8} = \frac{i}{1} = i$ $i = 0 + i$ in standard form.

Now work margin exercise 3.

Margin Exercise Answers

1. a. $28 + 4i$ b. $2 + 26i$ c. 3 2. a. $\frac{8}{5} + \frac{16}{5}i$ b. $\frac{1}{9} - \frac{4\sqrt{5}}{9}i$ c. $-\frac{1}{4} - i$
 d. $\frac{13}{19} - \frac{8\sqrt{3}}{19}i$ 3. a. i b. -1 c. 1

15.9 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

- When two complex numbers are multiplied, the _____ method can be used.
- When two complex conjugates are multiplied, the answer will always be a nonnegative _____.
- There are 4 possible values for any power of i : _____, _____, _____, and _____.
- The expressions $a + bi$ and $a - bi$ are _____ of each other.
- The product of $(a + bi)(a - bi)$ is _____.
- The expression $a + bi$ is called the _____ form of a complex number.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

- Regardless of the value of the exponent, the only possible values for any power of i are i and $-i$.
- The product $\sqrt{a} \cdot \sqrt{b}$ can be rewritten as \sqrt{ab} as long as a and b are real numbers.
- When i is squared, the product is 1.
- The conjugate of $4 - 5i$ is $4 + 5i$.

Practice

Perform the indicated operations and write each result in standard form. See Examples 1 and 2.

1. $8(2+3i)$
2. $-3(7-4i)$
3. $-7(\sqrt{2}-i)$
4. $\sqrt{3}(\sqrt{3}+2i)$
5. $3i(4-i)$
6. $-4i(6-7i)$
7. $-i(\sqrt{3}+i)$
8. $2i(\sqrt{5}+2i)$
9. $\sqrt{3}i(2-\sqrt{3}i)$
10. $5i(2-\sqrt{2}i)$
11. $(5+3i)(1+i)$
12. $(2+7i)(6+i)$
13. $(-3+5i)(-1+2i)$
14. $(6+2i)(3-i)$
15. $(2-3i)(2+3i)$
16. $(4+5i)(4-5i)$
17. $(4+3i)(7-2i)$
18. $(-2+5i)(i-1)$
19. $(5+7i)^2$
20. $(3+2i)^2$
21. $(\sqrt{3}+i)(\sqrt{3}-2i)$
22. $(2\sqrt{5}+3i)(\sqrt{5}-i)$
23. $(5-\sqrt{2}i)(5-\sqrt{2}i)$
24. $(\sqrt{7}+3i)(\sqrt{7}+i)$
25. $(4+\sqrt{5}i)(4-\sqrt{5}i)$
26. $(7+2\sqrt{3}i)(7-2\sqrt{3}i)$
27. $(\sqrt{5}+2i)(\sqrt{2}-i)$
28. $(2\sqrt{3}+i)(4+3i)$
29. $(3+\sqrt{5}i)(3+\sqrt{6}i)$
30. $(2-\sqrt{3}i)(3-\sqrt{2}i)$
31. $\frac{-3}{i}$
32. $\frac{7}{i}$
33. $\frac{5}{4i}$
34. $\frac{-3}{2i}$
35. $\frac{2+i}{-4i}$
36. $\frac{3-4i}{3i}$
37. $\frac{-4}{1+2i}$
38. $\frac{7}{5-2i}$
39. $\frac{6}{4-3i}$
40. $\frac{-8}{6+i}$
41. $\frac{2i}{5-i}$
42. $\frac{-4i}{1+3i}$
43. $\frac{2-i}{2+5i}$
44. $\frac{6+i}{3-4i}$
45. $\frac{2-3i}{-1+5i}$
46. $\frac{-3+i}{7-2i}$
47. $\frac{1+4i}{\sqrt{3}+i}$
48. $\frac{9-2i}{\sqrt{5}+i}$

49. $\frac{\sqrt{3} + 2i}{\sqrt{3} - 2i}$

50. $\frac{\sqrt{6} - 3i}{\sqrt{6} + 3i}$

Simplify the following powers of i and write each result in standard form. Assume k is a positive integer. See Example 3.

51. i^{13}

56. i^{-5}

52. i^{20}

57. i^{4k}

53. i^{30}

58. i^{4k+2}

54. i^{15}

59. i^{4k+3}

55. i^{-3}

60. i^{4k+1}

Find the indicated products and simplify.

61. $(x + 3i)(x - 3i)$

66. $(y - \sqrt{3}i)(y + \sqrt{3}i)$

62. $(y + 5i)(y - 5i)$

67. $[(x + 2) + 6i][(x + 2) - 6i]$

63. $(x + \sqrt{2}i)(x - \sqrt{2}i)$

68. $[(x + 1) - \sqrt{8}i][(x + 1) + \sqrt{8}i]$

64. $(2x + \sqrt{7}i)(2x - \sqrt{7}i)$

69. $[(y - 3) + 2i][(y - 3) - 2i]$

65. $(\sqrt{5}y + 2i)(\sqrt{5}y - 2i)$

70. $[(x - 1) + 5i][(x - 1) - 5i]$

Writing & Thinking

71. Explain why the product of every complex number and its conjugate is a nonnegative real number.
72. Explain why $\sqrt{-4} \cdot \sqrt{-4} \neq 4$. What is the correct value of $\sqrt{-4} \cdot \sqrt{-4}$?
73. What condition is necessary for the conjugate of a complex number, $a + bi$, to be equal to the reciprocal of this number?