Solution

a. $(6-2i)+(1-2i)=(6+1)+(-2-2)i=7-4i$

b. $(-8-\sqrt{2}\,i)-(-8+\sqrt{2}\,i)=(-8-(-8))+(-\sqrt{2}-\sqrt{2})i$

$=(-8+8)+(-2\sqrt{2})i$

$=0-2\sqrt{2}i$

$=-2\sqrt{2}\,i$ (or $-2i\sqrt{2}$)

c. $(\sqrt{3}-2i)+(1+\sqrt{5}\,i)=(\sqrt{3}+1)+(-2+\sqrt{5})\,i=(\sqrt{3}+1)+(\sqrt{5}-2)i$

Note: Here, the coefficients do not simplify. This means that the real part is $\sqrt{3}+1$ and the imaginary part is $\sqrt{5}-2$.

Now work margin exercise 4.

Margin Exercise Answers

1. a. $10i$ **b.** $7i$ **c.** $3i\sqrt{2}$ **d.** $6i\sqrt{2}$ **2. a.** real: 0; imaginary: 5 **b.** real: 14; imaginary: $\sqrt{7}$ **c.** real: $\frac{6}{5}$; imaginary: $-\frac{11}{5}$ **d.** real: -13; imaginary: 0 **3. a.** $x=10$ and $y=-2$ **b.** $y=1$ and $x=-5$ **4. a.** $7+4i$ **b.** $-4\sqrt{2}i$ **c.** $(4+\sqrt{2})+(8-\sqrt{5})i$

15.8 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. Adding and subtracting complex numbers is similar to adding and subtracting _____.
2. The square roots of negative numbers are not real numbers but are _____ numbers.
3. Complex numbers consist of two parts: a/an _____ part and a/an _____ part.
4. The standard form of a complex number is _____.
5. The expression $\sqrt{-a}$ can be rewritten as $\sqrt{a}\cdot\sqrt{-1}$ and _____.
6. The value i is defined to be _____.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. An irrational number is a real number.
8. Every real number is a complex number.
9. If $a+bi=c+di$, then $a=d$ and $b=c$.
10. The square root of negative one is one.

Practice

Find the real part and the imaginary part of each of the complex numbers. See Example 2.

1. $4-3i$

2. $\frac{3}{4}+i$

3. $-11+\sqrt{2}\,i$

4. $6+\sqrt{3}\,i$

5. $\frac{3}{8}$

6. $\frac{4}{7}i$

7. $\frac{4+7i}{5}$

8. $\frac{2-i}{4}$

9. $\frac{2}{3}+\sqrt{17}\,i$

10. $-\sqrt{5}+\frac{\sqrt{2}}{2}i$

Simplify each radical. See Example 1.

11. $\sqrt{-49}$

12. $\sqrt{-121}$

13. $-\sqrt{-64}$

14. $-\sqrt{-169}$

15. $\sqrt{147}$

16. $\sqrt{128}$

17. $2\sqrt{-150}$

18. $4\sqrt{-99}$

19. $-2\sqrt{-108}$

20. $2\sqrt{175}$

21. $\sqrt{242}$

22. $\sqrt{-192}$

23. $\sqrt{-1000}$

24. $\sqrt{-243}$

Solve the equations for x and y. See Example 3.

25. $x+3i=6-yi$

26. $2x-8i=-2+4yi$

27. $\sqrt{5}-2i=y+xi$

28. $\sqrt{2}-2yi=3x+6i$

29. $\sqrt{2}+i-3=x+yi$

30. $\sqrt{5}\,i-3+4i=x+yi$

31. $3x+2-7i=i-2yi+5$

32. $x+yi+8=2i+4-3yi$

33. $x+2i=5-yi-3-4i$

34. $2x+3+6i=7-yi-2i$

35. $2+3i+x=5-7i+yi$

36. $11i-2x+4=10-3i+2yi$

37. $2x-2yi+6=6i-x+2$

38. $x+4-3x+i=8+yi$

Find each sum or difference as indicated. See Example 4.

39. $(2+3i)+(4-i)$

40. $(7-i)+(3+6i)$

41. $(4+5i)-(3-2i)$

42. $(-3+2i)-(6+2i)$

43. $(4-3i)+(2-3i)$

44. $(7+5i)+(6-2i)$

45. $(8+9i)-(8-5i)$

46. $(-6+i)-(2+3i)$

47. $\left(\sqrt{5}-2i\right)+(3-4i)$

48. $(4+3i)-\left(\sqrt{2}+3i\right)$

49. $\left(7+\sqrt{6}\,i\right)+(-2+i)$

50. $\left(\sqrt{11}+2i\right)+(5-7i)$

51. $\left(\sqrt{3}+\sqrt{2}\,i\right)-\left(5+\sqrt{2}\,i\right)$

52. $\left(\sqrt{5}+\sqrt{3}\,i\right)+(1-i)$

53. $\left(5+\sqrt{-25}\right)-\left(7+\sqrt{-100}\right)$

54. $\left(1+\sqrt{-36}\right)-\left(-4-\sqrt{-49}\right)$

55. $\left(13-3\sqrt{-16}\right)+\left(-2-4\sqrt{-1}\right)$

56. $\left(7+\sqrt{-9}\right)-\left(3-2\sqrt{-25}\right)$

57. $(4+i)+(-3-2i)-(-1-i)$

58. $(-2-3i)+(6+i)-(2+5i)$

59. $(7+3i)+(2-4i)-(6-5i)$

60. $(-5+7i)+(4-2i)-(3-5i)$

Writing & Thinking

61. Answer the following questions and give a brief explanation of your answer.

a. Is every real number a complex number?

b. Is every complex number a real number?

62. List 5 numbers that do and 5 numbers that do not fit each of the following categories (if possible).

a. rational number

b. integer

c. real number

d. pure imaginary number

e. complex number

f. irrational number