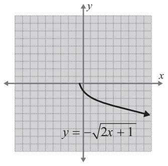
Note that you may need to adjust the window on your calculator. To adjust the window, use the following steps.

Press the WINDOW key and the standard window will be displayed. By default, the standard window displays a graph with x-values and y-values ranging from –10 to 10 with tic marks on the axes every 1 unit. This window can be changed at any time by changing the individual numbers or pressing the ZOOM key and selecting an option from the menu displayed. To return the screen back to the standard dimensions with x-values and y-values ranging from –10 to 10, press ZOOM and ZStandard. A square screen can be attained by pressing zoom and ZSquare or by pressing the window key and setting Xmin = –15 and Xmax = 15 to give the x-axis a length of 30 and the y-axis a length of 20 (a ratio of 3:2).

Margin Exercise Answers

1. a. $[-2,\infty)$ **b.** $(-\infty,\infty)$ **2. a.** $f(0)=0, f(2)=10, f(8)=20$ **b.** $f(1)=-1, f(2)=1, f(15)=3$

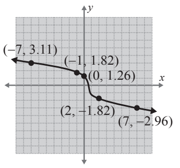
3.



4. a.

x	y_1
–7	3.1072
–1	1.817
0	1.2599
2	–1.817
7	–2.962

b.



15.7 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. A relation is a set of __________ pairs of real numbers.

2. The set of all second coordinates of a relation is the relation's _______.

3. The domain D of a relation is the set of all _____ __________ in the relation.

4. A/An ____________ is a relation in which each domain element has exactly one corresponding range element.

5. If any _____________ line intersects the graph of a relation at more than one point, then the relation is not a/an _____________.

6. A radical function is a function of the form $y=\sqrt[n]{g(x)}$ in which the radicand contains a/an ________ __________.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. If a radical function has an index that is an even number, then the domain is the set of all x such that $g(x)\ge 0$.

8. If a radical function has an odd numbered index, the domain is the set of all positive numbers.

9. Both the domain and the range of a radical function depend on the index.

10. To graph a radical function, you must be aware of its domain and you should plot at least a few points to see the nature of the resulting curve.

Practice

Find each function value as indicated and round decimal values to the nearest ten-thousandth, if necessary. See Example 2.

1. Given $f(x)=\sqrt{2x+1}$, find
   a. $f(2)$
   b. $f(4)$
   c. $f(24.5)$
   d. $f(1.5)$

2. Given $f(x)=\sqrt{5-3x}$, find
   a. $f(0)$
   b. $f(-2)$
   c. $f\left(-\frac{20}{3}\right)$
   d. $f(-2.4)$

3. Given $g(x)=\sqrt[3]{x+6}$, find
   a. $g(21)$
   b. $g(-7)$
   c. $g(-14)$
   d. $g(18)$

4. Given $h(x)=\sqrt[3]{4-x}$, find
   a. $h(4)$
   b. $h(-4)$
   c. $h(3.999)$
   d. $h(-2.5)$

Use interval notation to indicate the domain of each radical function. See Example 1.

5. $y=\sqrt{x+8}$

6. $y=\sqrt{2x-1}$

7. $y=\sqrt{2.5-5x}$

8. $y=\sqrt{1-3x}$

9. $f(x)=\sqrt[3]{x+4}$

10. $f(x)=\sqrt[3]{6x}$

11. $g(x)=\sqrt{6-3x}$

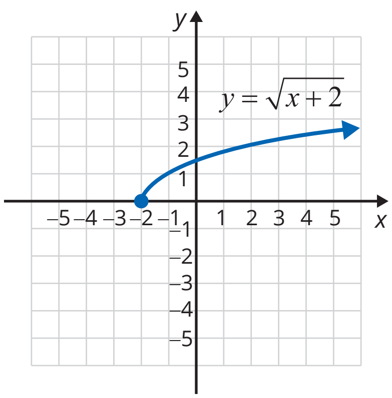
12. $g(x)=\sqrt{x+4}$

13. $y=\sqrt[3]{2-5x}$

14. $y=\sqrt[3]{5x+9}$

Identify the domain, range, and any zeros from the graphs of the following radical functions.

15.



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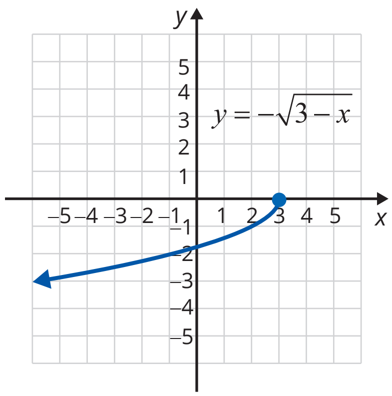

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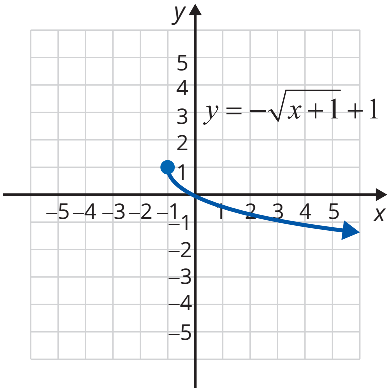

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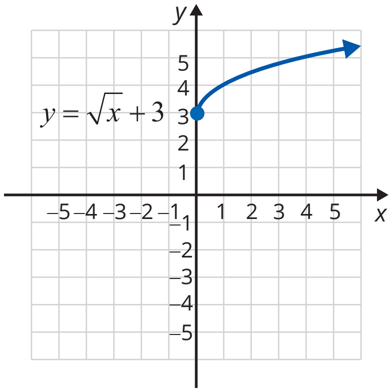

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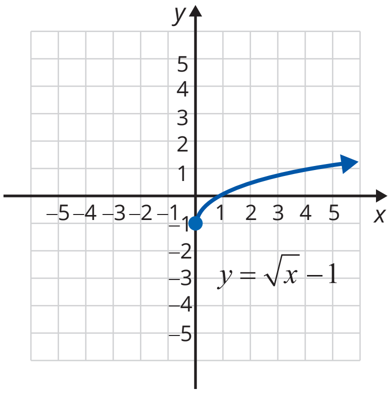

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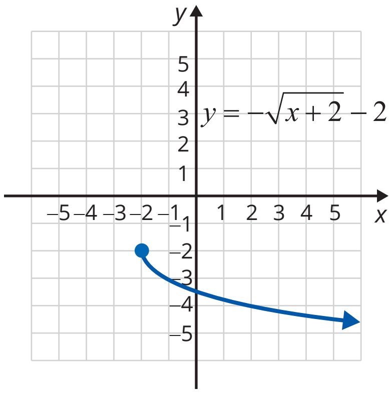


Match the functions given with the graphs of the functions (A) through (F).

21. $y=\sqrt{x-2}$

22. $y=\sqrt{2-x}$

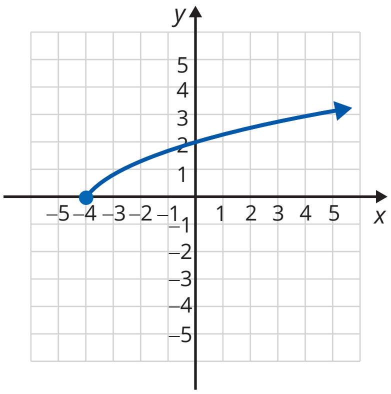
23. $y=-\sqrt{x-3}$

24. $y=-\sqrt{3-x}$

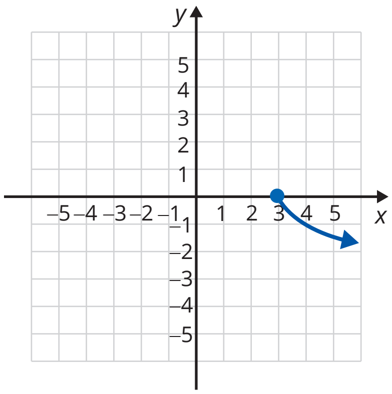
25. $y=\sqrt{x+4}$

26. $y=\sqrt{x-4}$

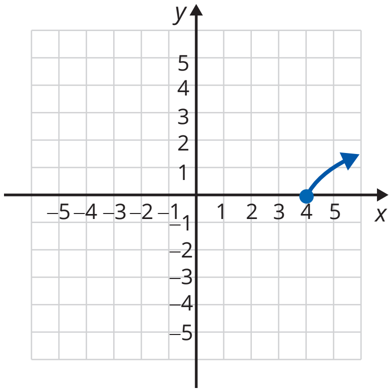
A.



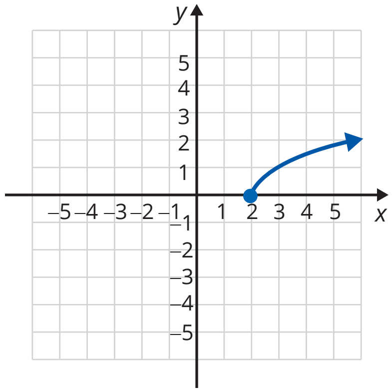
B.



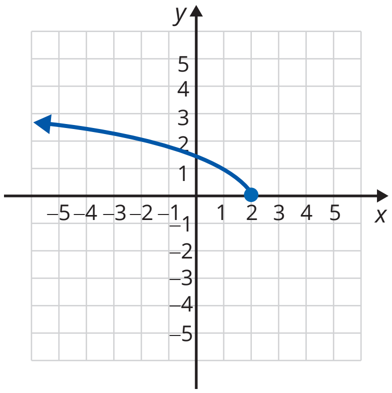
C.



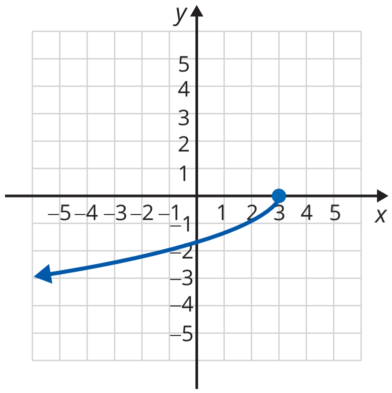
E.



D.



F.



Determine at least 5 points for the given function and then sketch the graph of the function. See Example 3.

27. $y=\sqrt{x-1}$

28. $y=\sqrt{2x+6}$

29. $f(x)=-\sqrt{3x+3}$

30. $h(x)=-\sqrt{x+1}$

31. $f(x)=\sqrt[3]{x+2}$

32. $g(x)=\sqrt[3]{x-6}$

33. $y=\sqrt[3]{3x+6}$

34. $y=\sqrt[3]{2x-4}$

Use a graphing calculator to graph each of the functions. See Example 4.

35. $y=3\sqrt{x+2}$

36. $y=2\sqrt{3-x}$

37. $g(x)=-\sqrt{2x}$

38. $f(x)=\sqrt{3x}$

39. $f(x)=-\sqrt{x+4}$

40. $f(x)=-\sqrt{5-x}$

41. $y=-\sqrt[3]{x+2}$

42. $y=-\sqrt[3]{3x+4}$

43. $g(x)=\sqrt[3]{2x}$

44. $y=\sqrt[3]{4-x}$

45. $y=\sqrt[3]{2x+1}$

46. $y=\sqrt[3]{x+7}$

Complete the following problems.

47. Graph the following three radical functions. For each function, state the domain of the function using interval notation. Then, describe how the graph of the function differs from the graph of $f(x)=\sqrt{x}$.

a. $f(x)=\sqrt{4x}$ **b.** $f(x)=\sqrt{x}-4$ **c.** $f(x)=-\sqrt{x-4}$

48. Graph the following three radical functions. For each function, state the domain of the function using interval notation. Then, describe how the graph of the function differs from the graph of $f(x)=\sqrt{x}$.

a. $f(x)=-\sqrt{x}$ **b.** $f(x)=\sqrt{x+1}$ **c.** $f(x)=\sqrt{x}+1$

49. Using your results from Exercises 47 and 48, discuss any general conclusions you can draw from the differences between $f(x)=\sqrt{x}$ and $f(x)=-\sqrt{(ax+b)}+c$.

Applications

Solve.

50. The hang time of an athlete can be represented by

$$t=2\sqrt{\frac{2h}{g}},$$

where t is the hang time of the athlete in seconds, h is the height of the jump in feet, and g is the acceleration due to gravity. (The gravity constant g can be estimated by using 32 ft/sec^2.)

a. Using a graphing calculator, graph this equation.

b. Identify the domain and range of this function. Does this make sense in the context of the function?

c. Find the hang time of an athlete with a 24-inch vertical leap. Round your answer to the nearest hundredth.

51. The motion of a simple pendulum is represented by the following equation, where T = the pendulum period in seconds, L = length in meters, and g = acceleration of gravity.

$$T=2\pi\sqrt{\frac{L}{g}}$$

Use $g = 9.8$ m/s^2 and $\pi = 3.14$.

a. Using a graphing calculator, graph this equation.

b. Identify the domain and range of this function. Does this make sense in the context of the function?

c. Find the pendulum period of a simple pendulum with length 15 meters. Round your answer to the nearest hundredth.

52. The relationship between the radius and volume of a cone of height 7 inches is as follows.

$$r = \sqrt{\frac{3V}{7\pi}}$$

In this equation, r is the radius of the cone in inches and V is the volume of the cone in cubic inches. Use $\pi = 3.14$.

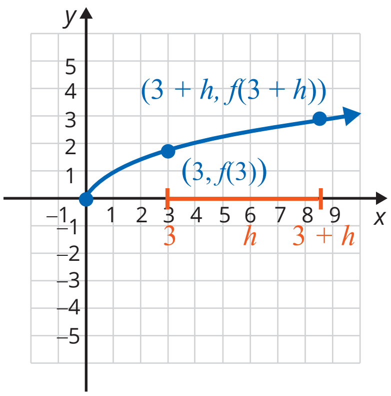
a. Using a graphing calculator, graph this equation.

b. Identify the domain and range of this function. Does this make sense in the context of the function?

c. Find the radius of a cone whose height is 7 inches and whose volume is 68 cubic inches. Round your answer to the nearest hundredth.

Writing & Thinking

53. The graph of the radical function $f(x) = \sqrt{x}$ is shown with two values of x on the x–axis, 3 and $3 + h$.



a. Rationalize the numerator of the expression $\dfrac{f(3+h)-f(3)}{h} = \dfrac{\sqrt{3+h}-\sqrt{3}}{h}$ by multiplying both the numerator and denominator by the conjugate of the numerator. Then simplify the resulting expression.

b. What do you think this expression represents graphically? (**Hint:** Two points determine a line.)

c. Using your results from parts a. and b., what do you see happening on the graph if the value of h shrinks slowly to 0?

d. Using your analysis from part c., what happens to the value of your simplified expression in part a. and what do you think this value represents?

54. Use your graphing calculator to graph the function $g(x) = \sqrt{x} \cdot \sqrt{x}$. Explain why the graph of this function differs from the graph of $f(x) = x$.