Completion Example Answers

2. a. $(3+5-1)\sqrt{6}=7\sqrt{6}$ **b.** $(1+8)\sqrt{a}+(4+3)\sqrt{b}=9\sqrt{a}+7\sqrt{b}$

4. a. $6\sqrt{3}\cdot\sqrt{3}+6\sqrt{3}\cdot 2\sqrt{7}=18+12\sqrt{21}$ **b.** $\sqrt{x}\cdot\sqrt{x}+\sqrt{x}\cdot(-4)+8\cdot\sqrt{x}+8\cdot(-4)=x+4\sqrt{x}-32$

Margin Exercise Answers

1. a. $9\sqrt{3a}$ **b.** $5\sqrt{5}+3\sqrt{3}$ **c.** $-\sqrt[3]{9x}$ **2. a.** $6\sqrt{7}$ **b.** $2\sqrt{x}-7\sqrt{3}$ **3. a.** 15 **b.** $13\sqrt{2}-13$ **c.** $8+3\sqrt{5}$ **d.** $3z-5$ **e.** $5x+5-4\sqrt{5x+1}$ **4. a.** $4\sqrt{30}-20$ **b.** $s+2\sqrt{s}-63$ **5. a.** 11.0711 **b.** 7

15.4 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. Like radicals have the same _____ and radicand or they can be simplified so that they do.
2. Sometimes two or more radicals that do not appear to be like radicals can be _____ so that they are like radicals.
3. To find the product of two binomials that contain radical terms, you can use the _____ method.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

4. The radicals $\sqrt{a}$ and $\sqrt[3]{a}$ are like radicals.
5. The radicals $3\sqrt{a}$ and $\sqrt{a}$ are like radicals.
6. The sum $4\sqrt{3}+8\sqrt{5}$ cannot be simplified.

Practice

Simplify the following radical expressions. Assume that all variables represent positive real numbers. See Examples 1 and 2.

1. $3\sqrt{2}+5\sqrt{2}$
2. $7\sqrt{3}-2\sqrt{3}$
3. $4\sqrt{11}-3\sqrt{11}$
4. $6\sqrt{5}+\sqrt{5}$
5. $8\sqrt{10}-11\sqrt{10}$
6. $6\sqrt{17}-9\sqrt{17}$
7. $4\sqrt[3]{3}+9\sqrt[3]{3}$
8. $11\sqrt[3]{14}-6\sqrt[3]{14}$
9. $6\sqrt{11}-5\sqrt{11}-2\sqrt{11}$
10. $\sqrt{7}+6\sqrt{7}-2\sqrt{7}$
11. $\sqrt{a}+4\sqrt{a}-2\sqrt{a}$
12. $2\sqrt{x}-3\sqrt{x}+7\sqrt{x}$
13. $5\sqrt{x}+3\sqrt{x}-\sqrt{x}$
14. $6\sqrt{xy}-10\sqrt{xy}+\sqrt{xy}$

15. $3\sqrt{2} + 5\sqrt{3} - 2\sqrt{3} + \sqrt{2}$

16. $\sqrt{5} + \sqrt{4} - 2\sqrt{5} + 6$

17. $2\sqrt{a} + 7\sqrt{b} - 6\sqrt{a} + \sqrt{b}$

18. $4\sqrt{x} - 3\sqrt{x} + 2\sqrt{y} + 2\sqrt{x}$

19. $6\sqrt[3]{x} - 4\sqrt[3]{y} + 7\sqrt[3]{x} + 2\sqrt[3]{y}$

20. $5\sqrt[3]{x} + 9\sqrt[3]{y} - 10\sqrt[3]{y} + 4\sqrt[3]{x}$

21. $\sqrt{12} + \sqrt{27}$

22. $\sqrt{32} - \sqrt{18}$

23. $3\sqrt{5} - \sqrt{45}$

24. $2\sqrt{7} + 5\sqrt{28}$

25. $3\sqrt[3]{54} + 8\sqrt[3]{2}$

26. $2\sqrt[3]{128} + 5\sqrt[3]{-54}$

27. $\sqrt{50} - \sqrt{18} - 3\sqrt{12}$

28. $2\sqrt{48} - \sqrt{54} + \sqrt{27}$

29. $2\sqrt{20} - \sqrt{45} + \sqrt{36}$

30. $\sqrt{18} - 2\sqrt{12} + 5\sqrt{2}$

31. $\sqrt{8} - 2\sqrt{3} + \sqrt{27} - \sqrt{72}$

32. $\sqrt{80} + \sqrt{8} - \sqrt{45} + \sqrt{50}$

33. $5\sqrt[3]{16} - 4\sqrt[3]{24} + \sqrt[3]{-250}$

34. $\sqrt[3]{192} - 2\sqrt[3]{128} + \sqrt[3]{-81}$

35. $6\sqrt{2x} - \sqrt{8x}$

36. $5\sqrt{3x} + 2\sqrt{12x}$

37. $5y\sqrt{2y} - y\sqrt{18y}$

38. $9x\sqrt{xy} - x\sqrt{16xy}$

39. $4x\sqrt{3xy} - x\sqrt{12xy} - 2x\sqrt{27xy}$

40. $x\sqrt{32x} - x\sqrt{50x} + 2x\sqrt{18x}$

41. $\sqrt{36x^3} + \sqrt{81x^3}$

42. $\sqrt{4a^2b} + \sqrt{9a^2b}$

43. $\sqrt{16x^3y^4} - \sqrt{25x^3y^4}$

44. $\sqrt{72x^{12}y^{15}} + \sqrt{18x^{12}y^{15}} + \sqrt{2x^{12}y^{15}}$

45. $\sqrt{12x^{10}y^{20}} + \sqrt{27x^{10}y^{20}} - \sqrt{3x^{10}y^{20}}$

46. $\sqrt[3]{8a^{12}} + \sqrt[3]{1000a^{12}}$

47. $\sqrt[3]{-27x^{24}y^6} + \sqrt[3]{-125x^{24}y^6}$

48. $\sqrt[3]{27a^{15}b} + \sqrt[3]{8a^{15}b} + \sqrt[3]{64a^{15}b}$

49. $\sqrt[3]{-16x^9y^{12}} - \sqrt[3]{16x^{12}y^9} + \sqrt[3]{54x^3y^6}$

50. $\sqrt[3]{54x^{13}y^3} + \sqrt[3]{8x^{23}y^6} + \sqrt[3]{3x^{13}y^3}$

Multiply the following radical expressions and then simplify the results. Assume that all variables represent positive real numbers. See Examples 3 and 4.

51. $\sqrt{2}\left(3 - 4\sqrt{2}\right)$

52. $2\sqrt{7}\left(\sqrt{7} + 3\sqrt{2}\right)$

53. $3\sqrt{18} \cdot \sqrt{2}$

54. $2\sqrt{10} \cdot \sqrt{5}$

55. $-2\sqrt{6} \cdot \sqrt{8}$

56. $2\sqrt{15} \cdot 5\sqrt{6}$

57. $\sqrt{3}\left(\sqrt{2} + 2\sqrt{12}\right)$

58. $\sqrt{2}\left(\sqrt{3} - \sqrt{6}\right)$

59. $\sqrt{y}\left(\sqrt{x} + 2\sqrt{y}\right)$

60. $\sqrt{x}\left(\sqrt{x} - 3\sqrt{y}\right)$

61. $\left(3+\sqrt{2}\right)\left(5-\sqrt{2}\right)$

62. $\left(\sqrt{6}+2\right)\left(\sqrt{6}-2\right)$

63. $\left(\sqrt{3x}-8\right)\left(\sqrt{3x}-1\right)$

64. $\left(6+\sqrt{2x}\right)\left(4+\sqrt{2x}\right)$

65. $\left(2\sqrt{7}+4\right)\left(\sqrt{7}-3\right)$

66. $\left(5\sqrt{3}-2\right)\left(2\sqrt{3}-7\right)$

67. $\left(\sqrt{5}+2\sqrt{2}\right)^2$

68. $\left(2\sqrt{5}+3\sqrt{2}\right)^2$

69. $\left(\sqrt{2}+\sqrt{3}\right)\left(\sqrt{5}-\sqrt{3}\right)$

70. $\left(\sqrt{6}+\sqrt{5}\right)\left(\sqrt{6}-\sqrt{2}\right)$

71. $\left(\sqrt{x}+\sqrt{6}\right)\left(\sqrt{x}-3\sqrt{6}\right)$

72. $\left(\sqrt{11}+\sqrt{3}\right)\left(\sqrt{11}-2\sqrt{3}\right)$

73. $\left(3\sqrt{7}+\sqrt{5}\right)\left(3\sqrt{7}-\sqrt{5}\right)$

74. $\left(7\sqrt{x}+\sqrt{2}\right)\left(7\sqrt{x}-\sqrt{2}\right)$

75. $\left(\sqrt{x}+5\sqrt{y}\right)^2$

76. $\left(3\sqrt{x}+\sqrt{y}\right)^2$

77. $\left(\sqrt{x+3}-5\right)^2$

78. $\left(\sqrt{x+2}+3\right)^2$

79. $\left(4-\sqrt{2x+3}\right)^2$

80. $\left(6-\sqrt{4x+1}\right)^2$

Use a graphing calculator to evaluate each expression. Round your answers to the nearest ten-thousandth, if necessary. See Example 5.

81. $13-\sqrt{75}$

82. $5-\sqrt{67}$

83. $\sqrt{900}+\sqrt{2.56}$

84. $\sqrt{1600}-\sqrt{1.69}$

85. $\left(\sqrt{7}+8\right)\left(\sqrt{7}-8\right)$

86. $\left(\sqrt{8}-\sqrt{5}\right)\left(\sqrt{8}+\sqrt{5}\right)$

87. $\left(2\sqrt{3}+5\sqrt{2}\right)\left(\sqrt{10}-3\sqrt{5}\right)$

88. $\left(6\sqrt{5}+5\sqrt{7}\right)\left(3\sqrt{2}-\sqrt{6}\right)$

Explain the error(s) made in each solution below.

89.
$$\begin{aligned}\sqrt{16}+\sqrt{48} &= \sqrt{16+48}\\ &= \sqrt{64}\\ &= 8\end{aligned}$$

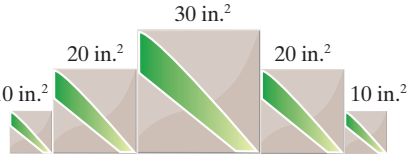
90.
$$\begin{aligned}\sqrt[3]{-125}+\sqrt[3]{98} &= \sqrt[3]{-125+98}\\ &= \sqrt[3]{-27}\\ &= 3\end{aligned}$$

Applications

Solve.

91. For a complete radio circuit, $d=\sqrt{2g}+\sqrt{2h}$, where d equals the visual horizon distance and g and h are the heights of the radio antennas at the respective stations. What is d when $g = 75$ ft and $h = 85$ ft?

92. Mary is making a tile decoration for her wall. Using square tiles of different sizes, Mary created one decoration that is five tiles across, with sides touching. The first tile is 10 in.^{2}, the second is 20 in.^{2}, the third is 30 in.^{2}, the fourth is 20 in.^{2}, and the fifth is 10 in.^{2} What is the length of the decoration?



93. Josue earns $\sqrt{32t}$ dollars when he works t hours. His roommate, Eric, earns $\sqrt{18t}$ dollars when he works t hours.

a. Find an expression that represents the total income that the two roommates earn after they each work t hours.

b. Find an expression that represents how much more money Josue will earn than Eric after they each work t hours.

94. The city planning committee is looking for places to build a community garden. One lot up for consideration has a length of $\frac{8+2\sqrt{b}}{a}$ and a width of $\frac{6+5\sqrt{b}}{a}$.

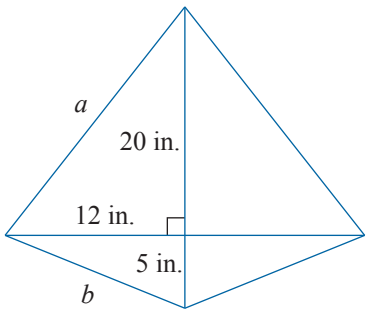
If $a = 3$ yards and $b = 7$ yards, what is the area of this lot? Leave your answer in simplified radical form.

95. The owner of an apple orchard has a field of new trees that are growing and producing more apples each year. He wants to determine the average growth rate, or percentage increase, in the amount of apples produced by these trees over time. The amount of growth varies each year, so he decides to find the geometric mean, or average. The formula $g = \sqrt{a} \cdot \sqrt{b}$ is used to find the geometric mean of two numbers.

a. Over two years, the growth rate was 180% and 120%. Find the average growth rate for these two years. Leave your answer in simplified radical form.

b. Over two years, the growth rate was $8x^3$% and $9x^5$%. Find the average growth rate for these two years. Leave your answer in simplified radical form.

96. A simple diamond-shaped kite is created from wooden dowel rods and nylon cloth. A diagram for the kite is shown.



a. Determine the length of side a. Write your answer in simplified radical form.

b. Determine the length of side b. Write your answer in simplified radical form.

c. Calculate the perimeter of the kite. Write your answer in simplified radical form.