15.1 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. In a radical expression that appears to have no index, the index is understood to be ___.
2. When a number is multiplied by itself, the product is said to be that number's _____.
3. To reverse the process of squaring, find the _____ _____ of the number.
4. If a is a nonnegative real number, then $\sqrt{a}$ is the _____ square root of a.
5. In $\sqrt{102}$, the symbol $\sqrt{\ }$ is called the _____ sign and 102 is the _____.
6. Square roots of negative numbers are not ____ numbers.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. If a number is squared and the principal square root of the result is found, that square root is always equal to the original number.
8. There is no real number that can be a square root of a negative number.
9. The index is the number underneath the radical sign.
10. The cube root of -27 is a real number.

Practice

Simplify the following square roots and cube roots. See Examples 1, 2, and 4.

1. $\sqrt{9}$
2. $\sqrt{49}$
3. $\sqrt{81}$
4. $\sqrt{36}$
5. $\sqrt{289}$
6. $\sqrt{121}$
7. $\sqrt{169}$
8. $\sqrt{361}$
9. $\sqrt[3]{1}$
10. $\sqrt[3]{1000}$
11. $\sqrt[3]{125}$
12. $\sqrt[3]{343}$
13. $\sqrt[3]{216}$
14. $\sqrt[3]{512}$
15. $\sqrt{\frac{1}{4}}$
16. $\sqrt{\frac{9}{16}}$
17. $\sqrt[3]{\frac{27}{64}}$
18. $\sqrt[3]{\frac{1}{8}}$
19. $\sqrt{0.04}$
20. $\sqrt{0.0081}$
21. $-\sqrt{100}$
22. $-\sqrt{144}$
23. $-\sqrt{0.0016}$
24. $-\sqrt{0.000004}$
25. $\sqrt[3]{-27}$
26. $\sqrt[3]{-64}$
27. $\sqrt[3]{-125}$
28. $\sqrt[3]{729}$

29. $\sqrt{\frac{9}{25}}$

30. $\sqrt{\frac{25}{81}}$

Estimates of radicals are given (rounded to the nearest ten-thousandth). Show that these are reasonable estimates. See Example 3.

31. $\sqrt{74} \approx 8.6023$

32. $\sqrt{18} \approx 4.2426$

33. $\sqrt{32} \approx 5.6569$

34. $\sqrt{110} \approx 10.4881$

In each of the following problems, determine the symbol, <, >, or =, that makes the statement true.

35. $\sqrt{16}$ ___ $\sqrt[3]{27}$

36. $\sqrt[3]{64}$ ___ $\sqrt{125}$

37. $\sqrt{36}$ ___ $\sqrt[3]{343}$

38. $\sqrt[3]{125}$ ___ $\sqrt{64}$

39. $\sqrt{4}$ ___ $\sqrt{4}$

40. $\sqrt{4}$ ___ $\sqrt[3]{8}$

41. $\sqrt[3]{343}$ ___ $\sqrt{49}$

42. $\sqrt{25}$ ___ $\sqrt[3]{27}$

Use your knowledge of square roots and cube roots to determine whether each number is rational, irrational, or not a real number.

43. $\sqrt{4}$

44. $\sqrt{17}$

45. $\sqrt{169}$

46. $\sqrt[3]{8}$

47. $\sqrt{\frac{2}{9}}$

48. $-\sqrt{\frac{1}{4}}$

49. $\sqrt{-36}$

50. $\sqrt[3]{-27}$

51. $-\sqrt[3]{125}$

52. $\sqrt{-10}$

53. $\sqrt{1.68}$

54. $\sqrt{5.29}$

Use a calculator to find the value of each radical expression rounded to the nearest ten-thousandth. See Example 5.

55. $\sqrt{39}$

56. $\sqrt{150}$

57. $\sqrt{6.23}$

58. $\sqrt{9.6}$

59. $\sqrt{\frac{1}{5}}$

60. $\sqrt{\frac{3}{8}}$

61. $4\sqrt{5}$

62. $6\sqrt{3}$

63. $-2\sqrt{17}$

64. $-3\sqrt{6}$

65. $\sqrt[3]{18}$

66. $\sqrt[3]{26}$

67. $2\sqrt[3]{15}$

68. $3\sqrt[3]{11}$

Applications

Use the formula $s = \sqrt[3]{V}$, which relates the length of the sides of a cube and the volume V, to answer the following questions.

69. The volume of a puzzle cube is 250 cubic inches. What is the length of one side? Round your answer to the nearest hundredth.

70. Three cubic blocks of different volumes were stacked on top of each other. The top block was 216 cubic centimeters. The middle block was 343 cubic centimeters, and the bottom block was 512 cubic centimeters. How tall was the stack of blocks?

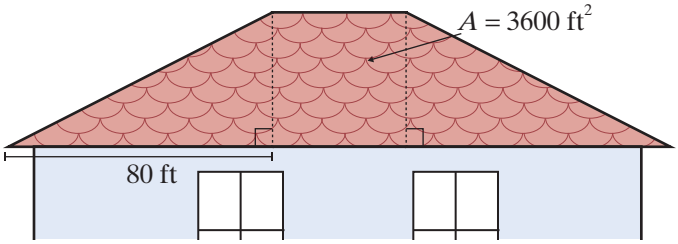
Solve.

71. The area of a square tile is 16 square inches.

a. How long are the sides of the square tile?

b. How many tiles would be needed for a four-foot-long and four-inch-high backsplash in a newly designed bathroom?

72. Margaret needs to put a new gutter on one side of her roof. The shape of her roof is made up of two right triangles that are on opposite sides of a square. If the area of the square is 3600 ft^2 and the base of each of the triangles is 80 ft, what is the total length of the gutter she'll need to replace?



73. The volume of a volleyball is 523.33 cubic inches. Find the diameter of the ball using $\pi = 3.14$. Round your answer to the nearest inch. (**Note:** For a sphere, $V = \frac{4}{3}\pi r^3$.)

74. The volume of a child's building block is 64 cubic centimeters.

a. Assuming the building block is a perfect cube, find the length of each side of the block.

b. If a child stacks 5 blocks directly on top of each other, find the height of the structure that is created.

75. A pastry chef is making a batch of mini *petit fours*, which are little cakes, in the shape of cubes. To keep the nutritional value of each *petit four* consistent, the bakery manager wants each one to have a volume of 100 cm^3. What should the side length be, to the nearest hundredth, for each *petit four*? (**Note:** For volume of a cube, $V = s^3$ where s = side length.)

76. Isaac Newton fell asleep under an apple tree thinking about math. While he was sleeping, a squirrel knocked an apple off of a branch of the tree. The equation $t = \sqrt{\frac{2d}{9.8}}$ can be used to find the time t in seconds it takes for the apple to drop a certain distance d, where d is in meters. Round all answers to the nearest hundredth.

a. If the apple was connected to a branch 2 m above Newton's head, how long would it take before the apple hit Newton's head?

b. If the squirrel knocked a second apple off a branch that was 5 m above Newton's head, how long would it take before the apple hit Newton's head?

c. Suppose the second apple missed Newton's head and landed on the ground instead. If Newton's head was 0.8 m above the ground, how long would it take for the apple to hit the ground?

77. A person's Body Mass Index (BMI) is determined by the formula $B = \frac{m}{h^2}$ where B is the BMI, m is the person's mass in kilograms, and h is the person's height in meters. Having a BMI between 18.5 and 25 is considered optimal. To find a person's height based on their BMI and mass, the formula can be rearranged to $h = \sqrt{\frac{m}{B}}$. Round all answers to the nearest tenth.

a. Elias has a mass of 60.7 kg and a BMI of 21. What is Elias's height?

b. Fatima has a mass of 69.0 kg and a BMI of 27. What is Fatima's height?

c. Tobias has a mass of 69.0 kg and a BMI of 21. What is Tobias's height?

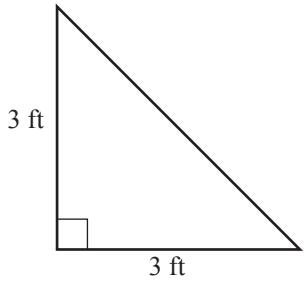
78. A square flower garden covers an area of 68 square feet.

a. What is the approximate length of each side of the square? Round your answer to the nearest tenth.

b. Use the answer from part a. to determine the amount of edging material needed to create a border around the flower garden.

c. The edging material costs $1.39 per foot. How much will the amount of edging material from part b. cost? Round your answer to the nearest cent.

79. Barbara's Bombtastic Bakery is installing a corner display stand for custom decorated cakes. The top of the display stand is designed in the shape of a right triangle, as shown.



a. What is the length of the longest side of the display stand? Round your answer to the nearest tenth.

b. The top of the display has a decorative edge around all three sides to prevent the cakes from falling off. How much edging is required for the display?

c. If the top of the display stand is to be covered by square tiles that have a side length of 6 inches, how many tiles will be needed?

80. A glass company makes a paperweight in the shape of a cube that has a volume of 91.125 cubic inches.

a. What is the length of each side of the cube?

b. What is the area of the base of the cube?

c. What is the surface area of the cube?

Writing & Thinking

81. Discuss, in your own words, why the square root of a negative number is not a real number.

82. Discuss, in your own words, why the cube root of a negative number is a negative number.