$$x^2 - 4x - 60 = 0$$
$$(x-10)(x+6) = 0$$
$$x - 10 = 0 \quad \text{or} \quad x + 6 = 0$$
$$x = 10 \qquad\qquad x = -6$$

Because the length of a side cannot be negative, the only acceptable solution is $x = 10$. Thus, $\overline{QR} = 10$. Substituting 10 for $x$ gives $\overline{AB} = 10 - 4 = 6$.

***Now work margin exercise 8.***

***Completion Example Answers***

Restrictions: $x \neq -5, 1$

**6.** $$(x+5)(x-1)\cdot\frac{x}{x-1} - (x+5)(x-1)\cdot\frac{3x+1}{(x+5)(x-1)} = (x+5)(x-1)\cdot\frac{x+2}{x+5}$$
$$(x+5)\cdot x - (3x+1) = (x-1)(x+2)$$
$$x^2 + 5x - 3x - 1 = x^2 + x - 2$$
$$2x - 1 = x - 2$$
$$x = -1$$

***Margin Exercise Answers***

**1. a.** $x = 11$ **b.** $x = 16$ **2.** 225 miles **3.** $x \neq -7, -2, 0$; $x = -4, 3$ **4.** $x \neq 0, 2$; $x = \frac{2}{5}$

**5.** $x \neq -5, 0, 5$; no solution **6.** $y \neq -5, 2$; $y = \frac{1}{7}$ **7.** $l = \frac{SA - 2wh}{2h + 2w}$ **8.** $x = 9$

# 14.6 Exercises

## Concept Check

**Fill-in-the-Blank.** Complete each sentence using information found in this section.

1. A comparison of two numbers by division is called a/an ________.
2. An equation stating that two ratios are equal is called a/an ________.
3. Solutions that are not actually solutions of the original equation are called __________ solutions.
4. One method of solving proportions is to clear the equation of ______ by first multiplying both sides of the equation by the LCD.
5. Rational expressions may contain ______ in either the numerator, the denominator, or both.
6. When solving an equation containing rational expressions, multiply both sides of the equation by the LCD and ______.

**True/False.** Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. An equation that involves the sum of rational expressions is also a proportion.
8. Multiplying by an LCD can cause extraneous roots.
9. A proportion is properly written if the numerators agree in type and the denominators agree in type.
10. When checking the solutions in the original equation, any solution that gives a 0 denominator cannot be checked.

## Practice

State any restrictions on *x*, and then solve the proportions. See Example 1.

1. $\frac{4x}{7}=\frac{x+5}{3}$
2. $\frac{3x+1}{4}=\frac{2x+1}{3}$
3. $\frac{10}{x}=\frac{5}{x-2}$
4. $\frac{8}{x-3}=\frac{12}{2x-3}$
5. $\frac{4}{x-4}=\frac{2}{x+3}$
6. $\frac{3}{x+5}=\frac{6}{x-2}$
7. $\frac{x+2}{5x}=\frac{x-6}{3x}$
8. $\frac{x-4}{3x}=\frac{x-2}{5x}$
9. $\frac{5x+2}{x-6}=\frac{11}{4}$
10. $\frac{x+9}{3x+2}=\frac{5}{8}$

State any restrictions on *x*, and then solve the equations. See Examples 3 through 6.

11. $\frac{5x}{4}-\frac{1}{2}=-\frac{3}{16}$
12. $\frac{x}{6}-\frac{1}{42}=\frac{1}{7}$
13. $\frac{3x-1}{6}-\frac{x+3}{4}=\frac{7}{12}$
14. $\frac{x-2}{3}-\frac{x-3}{5}=\frac{13}{15}$
15. $\frac{2+x}{4}-\frac{5x-2}{12}=\frac{8-2x}{5}$
16. $\frac{4x+1}{5}=\frac{2x+3}{2}-\frac{x+2}{4}$
17. $\frac{2}{3x}=\frac{1}{4}-\frac{1}{6x}$
18. $\frac{1}{x}-\frac{8}{21}=\frac{3}{7x}$
19. $\frac{3}{5x}-\frac{1}{5}=\frac{3}{4x}$
20. $\frac{3}{8x}-\frac{7}{10}=\frac{1}{5x}$
21. $\frac{3}{4x}-\frac{1}{2}=\frac{7}{8x}+\frac{1}{6}$
22. $\frac{5}{3x}+\frac{1}{2}=\frac{7}{9x}-\frac{5}{6}$
23. $\frac{2}{4x+1}=\frac{4}{x^2+9x}$
24. $\frac{3}{4x-1}=\frac{4}{x^2+x}$
25. $\frac{9}{x^2-6x}=\frac{5}{2x-3}$
26. $\frac{-9}{x^2+5x}=\frac{8}{4-9x}$
27. $\frac{x}{x-4}-\frac{4}{2x-1}=1$
28. $\frac{x}{x+3}+\frac{1}{x+2}=1$

**29.** $\frac{x+2}{x+1}+\frac{x+2}{x+4}=2$

**30.** $\frac{3x-2}{x+4}+\frac{2x+5}{x-1}=5$

**31.** $\frac{2}{4x-1}+\frac{1}{x+1}=\frac{3}{x+1}$

**32.** $\frac{x-2}{x+4}-\frac{3}{2x+1}=\frac{x-7}{x+4}$

**33.** $\frac{x-2}{x-3}+\frac{x-3}{x-2}=\frac{2x^2}{x^2-5x+6}$

**34.** $\frac{x}{x-4}-\frac{12x}{x^2+x-20}=\frac{x-1}{x+5}$

**35.** $\frac{3x+5}{3x+2}+\frac{8x+16}{3x^2-4x-4}=\frac{x+2}{x-2}$

**36.** $\frac{3x+5}{3x+2}-\frac{4-2x}{3x^2+8x+4}=\frac{x+4}{x+2}$

**37.** $\frac{3}{3x-1}+\frac{1}{x+1}=\frac{4}{2x-1}$

**38.** $\frac{2}{x+1}+\frac{4}{2x-3}=\frac{4}{x-5}$

Solve each of the formulas for the specified variable. Assume no denominator has a value of 0. See Example 7.

**39.** $S=\frac{a}{1-r}$; solve for $r$ (formula for the sum of an infinite geometric sequence)

**40.** $z=\frac{x-\overline{x}}{s}$; solve for $x$ (formula used in statistics)

**41.** $z=\frac{x-\overline{x}}{s}$; solve for $s$ (formula used in statistics)

**42.** $a_n=a_1+(n-1)d$; solve for $d$ (formula for the $n^{\text{th}}$ term in an arithmetic sequence)

**43.** $m=\frac{y-y_1}{x-x_1}$; solve for $y$ (formula for the slope of a line)

**44.** $v_{\text{avg}}=\frac{d_2-d_1}{t_2-t_1}$; solve for $d_2$ (formula for mean velocity)

**45.** $\frac{1}{R_{\text{total}}}=\frac{1}{R_1}+\frac{1}{R_2}$; solve for $R_{\text{total}}$ (formula used in electronics)

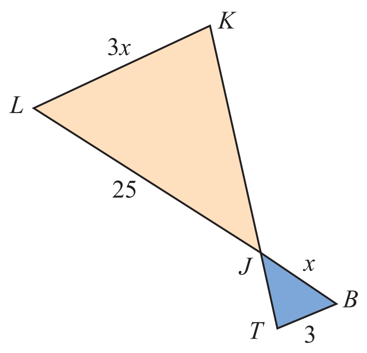
**46.** $\frac{1}{x}=\frac{1}{t_1}+\frac{1}{t_2}$; solve for $x$ (formula used in mathematics)

**47.** $A=P+Pr$; solve for $P$ (formula used for compound interest)

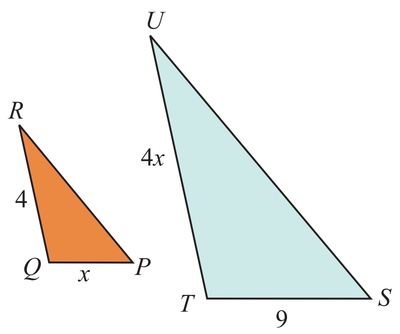
**48.** $y=\frac{ax+b}{cx+d}$; solve for $x$ (formula used in mathematics)

The following exercises show pairs of similar triangles. Find the lengths of the sides labeled with variables. See Example 8.

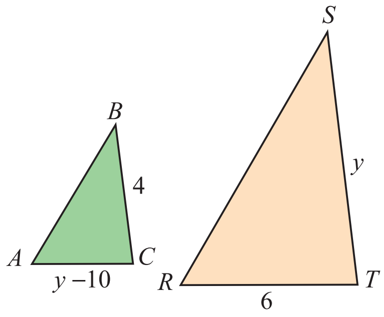
**49.** $\Delta JKL \sim \Delta JTB$



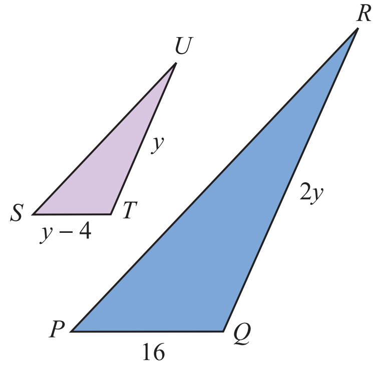

**50.** $\Delta QRP \sim \Delta TUS$



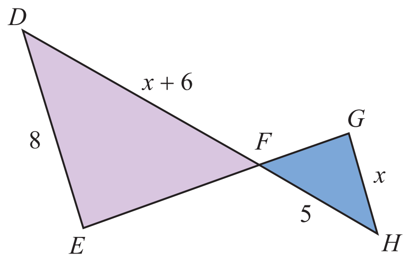
**51.** $\Delta ABC \sim \Delta RST$



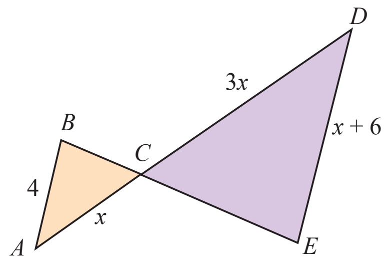

**53.** $\Delta SUT \sim \Delta PRQ$



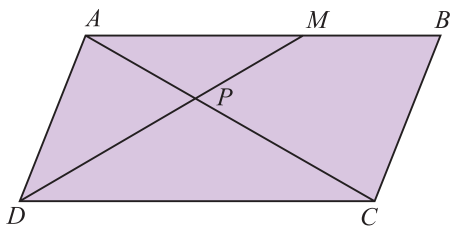

**52.** $\Delta FED \sim \Delta FGH$



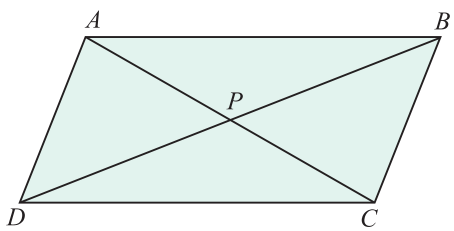

**54.** $\Delta ABC \sim \Delta DEC$




**55.** In the parallelogram *ABCD*, *AB* = *CD* = 10 in. Diagonal *AC* = 12 in. The point *M* on $\overline{AB}$ is 6 in. from *A*. Point *P* is the intersection of $\overline{DM}$ with $\overline{AC}$. The triangles *APM* and *CPD* are similar. (Symbolically, $\Delta APM \sim \Delta CPD$.) What are the lengths of $\overline{AP}$ and $\overline{PC}$?




**56.** If, in the same parallelogram discussed in Exercise 55, the point *P* is the point of intersection of the two diagonals, $\overline{AC}$ and $\overline{DB}$, what are the lengths of $\overline{AP}$ and $\overline{PC}$?



## Applications

Solve.

**57.** Making a statistical analysis, Ana found 3 defective computers in a sample of 20 computers. If this ratio is consistent, how many defective computers does she expect to find in a batch of 2400 computers?

**58.** At the Bright-As-Day light bulb plant, 3 out of each 100 bulbs produced are defective. If the daily production is 4800 bulbs, how many are defective?

**59.** A university has a ratio of 1 professor for every 23 students. If there are 1600 faculty members at the university, how many students are enrolled there?

**60.** New York Yankees player Didi Gregorius has a recorded batting average of 15 hits for every 50 times at bat. If he maintains this average, how many at-bats will he need to achieve 111 hits? (Round to the nearest whole number.)

**61.** On a map of Maryland, one inch represents 4 miles. If there are 8.5 inches between Baltimore, MD, and Washington, DC, how far are the two cities from each other?

**62.** A floor plan is drawn to scale in which 1 inch represents 4 feet. What size will the drawing be for a room that is 30 feet by 40 feet? (**Hint:** Set up two proportions.)

**63.** The recipe for Nestle Tollhouse Chocolate Chip Cookies calls for 2 cups of chocolate chips to make 5 dozen cookies. If you want to bake 17 dozen cookies, how many cups of chocolate chips do you need?

**64.** The instructions for Never-Ice Antifreeze states that 4 quarts of antifreeze are needed for every 10 quarts of radiator capacity. If Sal's car has a 22-quart radiator, how many quarts of antifreeze will it need?

**65.** An architect is to draw plans for a city park. He intends to use a scale of $\frac{1}{2}$ inch to represent 25 feet. How many inches will be needed to use for the length and width of a rectangular playing field that is 50 yards by 125 yards? (**Note:** 1 yard = 3 feet.)

**66.** A test driver wants to increase the speed of the car he is driving by 3 miles per hour every 2 seconds. However, he can only check his speed every 5 seconds because he is busy with other tasks during the test drive.

**a.** By how much should he increase his speed in 5 seconds?

**b.** If he starts checking his speed at 40 miles per hour, how fast should he be going after 10 seconds?

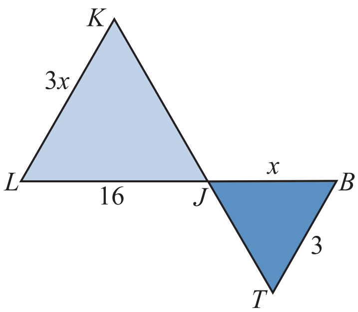
**67.** Jack and Diane are decorating a nursery room for their baby, which will be born in a few months. In one hour, Jack can get $\frac{1}{6}$ of the nursery done and Diane can get $\frac{1}{12}$ of the nursery done. If they work together, they can get $\frac{1}{x}$ of the nursery done in one hour. Determine how many hours it will take Jack and Diane to decorate the nursery if they work together by solving the equation $\frac{1}{6}+\frac{1}{12}=\frac{1}{x}$ for $x$.

**68.** A local print shop has a big order of pamphlets to print, so they decide to use two of their printers for the one job. The newer printer can print the pamphlets four times as fast as the older printer. That means in one hour, the newer printer can complete $\frac{1}{x}$ of the print job and the older printer can complete $\frac{1}{4x}$ of the print job. Working together, the printers can complete the job in 4 hours. Determine how many hours it would take the newer printer to print all of the pamphlets by itself by solving the equation $\frac{1}{x}+\frac{1}{4x}=\frac{1}{4}$ for $x$.

**69.** Two groups of civil engineers are surveying an area to prepare for the construction of a shopping center. The first group is full of new college graduates, and it will take them four more hours than it takes the second group, which is full of seasoned professionals. The second group can complete the job in $x$ hours. This means that in one hour, the first group can complete $\frac{1}{x+4}$ of the job and the second group can complete $\frac{1}{x}$ of the job. Working together, they can complete the surveying job in $\frac{15}{4}$ hours. Determine how many hours it would take each team to complete the job individually by solving the equation $\frac{1}{x+4}+\frac{1}{x}=\frac{4}{15}$ for $x$.

**70.** Terrence and Alicia are competing in a marathon where the average running speed is $x$ kilometers per hour. Terrence is running 2 kilometers per hour slower than the average running speed. Alicia is running 2 kilometers per hour faster than the average running speed. After a certain amount of time, Terrence ran 4 kilometers and Alicia ran 6 kilometers.

- **a.** Determine the speed of the average runner by solving the equation $\frac{4}{x-2}=\frac{6}{x+2}$ for $x$.
- **b.** What was Terrence's average running speed?
- **c.** What was Alicia's average running speed?
- **d.** How long did it take Terrence to run 4 kilometers and Alicia to run 6 kilometers?




**71.** A team of gardeners is making two flower beds that are in the shape of similar triangles outside of an art museum. The apprentice gardener wasn't completely paying attention to the instructions given by the master gardener. All that he can remember is that the flower beds are isosceles triangles, the base of the small triangle is 3 feet wide, one side of the larger triangle is 16 feet long, and the base of the large triangle is three times the side length of the small triangle. The apprentice gardener needs to determine the unknown dimensions of the triangles.

- **a.** Use the figure to write an equation to show that the side lengths are proportional.
- **b.** Solve the equation from part a. for $x$.
- **c.** Do any of the solutions from part b. not make sense in the context of the problem? If yes, explain why.
- **d.** What are the lengths of the unknown sides of the triangles?

## Writing & Thinking

In simplifying rational expressions, the result is a rational or polynomial expression. However, in solving equations with rational expressions, the goal is to find a value (or values) for the variable that will make the equation a true statement. Many students confuse these two ideas. To avoid confusing the techniques for adding and subtracting rational expressions with the techniques for solving equations, simplify the expression in part a. and solve the equation in part b. Explain, in your own words, the differences in your procedures. Assume no denominator has a value of 0.

**72.** **a.** $\frac{10}{x}+\frac{31}{x-1}+\frac{4x}{x-1}$

**b.** $\frac{10}{x}+\frac{31}{x-1}=\frac{4x}{x-1}$

**73.** **a.** $\frac{-4}{x^2-16}+\frac{x}{2x+8}-\frac{1}{4}$

**b.** $\frac{-4}{x^2-16}+\frac{x}{2x+8}=\frac{1}{4}$

**74.** **a.** $\frac{3x}{x^2-4}+\frac{5}{x+2}+\frac{2}{x-2}$

**b.** $\frac{3x}{x^2-4}+\frac{5}{x+2}=\frac{2}{x-2}$

**75.** **a.** $\frac{7}{5x}+\frac{2}{x-4}-\frac{3}{5x}$

**b.** $\frac{7}{5x}+\frac{2}{x-4}=\frac{3}{5x}$

**76.** **a.** $\frac{2}{x+9}-\frac{2}{x-9}+\frac{1}{2}$

**b.** $\frac{2}{x+9}-\frac{2}{x-9}=\frac{1}{2}$