Solution

a.
$$\begin{aligned} x^3 - 8 &= x^3 - 2^3 \\ &= (x-2)(x^2 + 2\cdot x + 2^2) \\ &= (x-2)(x^2 + 2x + 4) \end{aligned}$$

Note: Remember that the second polynomial is not a perfect square trinomial and cannot be factored.

b.
$$\begin{aligned} x^6 + 64y^3 &= (x^2)^3 + (4y)^3 \\ &= (x^2 + 4y)\left[(x^2)^2 - 4y\cdot x^2 + (4y)^2\right] \\ &= (x^2 + 4y)(x^4 - 4x^2y + 16y^2) \end{aligned}$$

c. Factor out the GCF first. Then factor the **difference of two cubes**.

$$\begin{aligned} 16y^{12} - 250 &= 2(8y^{12} - 125) \\ &= 2\left[(2y^4)^3 - 5^3\right] \\ &= 2(2y^4 - 5)\left[(2y^4)^2 + (5)(2y^4) + 5^2\right] \\ &= 2(2y^4 - 5)(4y^8 + 10y^4 + 25) \end{aligned}$$

Now work margin exercise 4.

Margin Exercise Answers

1. a. $7a(x-7)(x+7)$ **b.** $(y^3+10)(y^3-10)$ **2. a.** Not factorable **b.** $5(9x^2+4)$

3. a. $(z+20)^2$ **b.** $(y-7)^2$ **c.** $3z(x-3y)^2$ **d.** $(y+4-z)(y+4+z)$

4. a. $(y-3)(y^2+3y+9)$ **b.** $(2y-x^2)(4y^2+2x^2y+x^4)$ **c.** $6(2x^4-5)(4x^8+10x^4+25)$

13.4 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. Factoring a perfect square trinomial gives a square ___________.
2. In a perfect square trinomial, both the first and last terms must be perfect ______.
3. If the first term of a perfect square trinomial is x^2, and the last term is of the form a^2, then the middle term must be of the form _____ or _____.
4. The formula for factoring the difference of cubes is $x^3 - a^3 =$ ___________.
5. The formula for factoring the sum of two cubes is $x^3 + a^3 =$ ___________.
6. The first 6 perfect cubes are 1, 8, ___, ___, ___, and ___.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. The expression $x^2 + 20x + 100$ is a perfect square trinomial.

8. When factoring polynomials, always look for a common monomial factor first.

9. The sum of two squares, $(x^2 + a^2)$, is factorable.

10. Sixty-four is a perfect square and a perfect cube.

Practice

Completely factor each of the given polynomials. If a polynomial cannot be factored, write "not factorable." See Examples 1 through 4.

1. $x^2 - 25$

2. $y^2 - 121$

3. $81 - y^2$

4. $25 - z^2$

5. $2x^2 - 128$

6. $3x^2 - 147$

7. $4x^4 - 64$

8. $5x^4 - 125$

9. $y^2 + 100$

10. $4x^2 + 49$

11. $y^2 - 16y + 64$

12. $z^2 + 18z + 81$

13. $-4x^2 + 100$

14. $-12x^4 + 3$

15. $9x^2 - 25$

16. $4x^2 - 49$

17. $y^2 - 10y + 25$

18. $x^2 + 12x + 36$

19. $4x^2 - 4x + 1$

20. $49x^2 - 14x + 1$

21. $25x^2 + 30x + 9$

22. $9y^2 + 12y + 4$

23. $16x^2 - 40x + 25$

24. $9x^2 - 12x + 4$

25. $4x^3 - 64x$

26. $50x^3 - 8x$

27. $2x^3y + 32x^2y + 128xy$

28. $3x^2y - 30xy + 75y$

29. $y^2 + 6y + 9$

30. $y^2 + 4y + 4$

31. $x^2 - 20x + 100$

32. $25x^2 - 10x + 1$

33. $x^4 + 10x^2y + 25y^2$

34. $16x^4 + 8x^2y + y^2$

35. $x^3 - 125$

36. $x^3 - 64$

37. $y^3 + 216$

38. $y^3 + 1$

39. $x^3 + 27y^3$

40. $8x^3 + 1$

41. $x^2 + 64y^2$

42. $3x^3 + 81$

43. $4x^3 - 32$

44. $64x^3 + 27y^3$

45. $54x^3 - 2y^3$

46. $3x^4 + 375xy^3$

47. $x^3y + y^4$

48. $x^4y^3 - x$

49. $x^2y^2 - x^2y^5$

50. $2x^2 - 16x^2y^3$

51. $24x^4y + 81xy^4$

52. $x^6 - 64y^3$

53. $x^6 - y^9$

54. $64x^2 + 1$

55. $27x^3 + y^6$

56. $x^3 + 64z^3$

57. $8x^3 + y^3$

58. $x^3 + 125y^3$

59. $8y^3 - 8$

60. $36x^3 + 36$

61. $9x^2 - y^2$

62. $x^2 - 4y^2$

63. $x^4 - 16y^4$

64. $81x^4 - 1$

65. $(x-y)^2 - 81$

66. $(x+2y)^2 - 25$

67. $(x^2 - 2xy + y^2) - 36$

68. $(x^2 + 4xy + 4y^2) - 25$

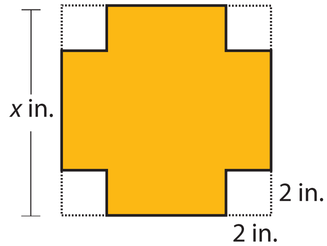
69. $(16x^2 + 8x + 1) - y^2$

70. $x^2 - (y^2 + 6y + 9)$

Solve.

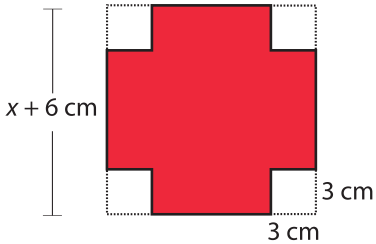
71. a. Represent the area of the shaded region of the square shown below as the difference of two squares.

b. Use the factors of the expression in part a. to draw (and label the sides of) a rectangle that has the same area as the shaded region.



72. a. Use a polynomial function to represent the area of the shaded region of the square.

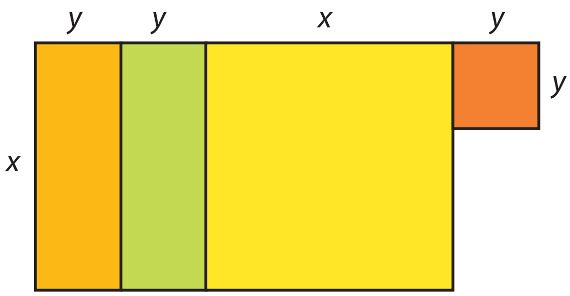
b. Use a polynomial function to represent the perimeter of the shaded figure.



Writing & Thinking

73. a. Show that the sum of the areas of the rectangles and squares in the figure is a perfect square trinomial.

b. Rearrange the rectangles and squares in the form of a square and represent its area as the square of a binomial.



74. Compound interest is interest earned on interest. If a principal P is invested and compounded annually (once a year) at a rate of r, then the amount, A_1 accumulated in one year is $A_1 = P + Pr$.
In factored form, we have $A_1 = P + Pr = P(1+r)$.
At the end of the second year the amount accumulated is $A_2 = (P+Pr)+(P+Pr)r$.

- **a.** Write the expression for A_2 in factored form similar to that for A_1.
- **b.** Write an expression for the amount accumulated in three years, A_3, in factored form.
- **c.** Write an expression for A_n the amount accumulated in n years.
- **d.** Use the formula you developed in part c. and your calculator to find the amount accumulated if \$10,000 is invested at 6% and compounded annually for 20 years.

75. You may have heard of (or studied) the following rules for division of an integer by 3 and 9:

1. An integer is divisible by 3 if the sum of its digits is divisible by 3.
2. An integer is divisible by 9 if the sum of its digits is divisible by 9.

The proofs of both **1.** and **2.** can be started as follows.

Let abc represent a three-digit integer.

$$\begin{aligned}\text{Then } abc &= 100a + 10b + c \\ &= (99+1)a + (9+1)b + c \\ &= (\text{now you finish the proofs})\end{aligned}$$

Use the pattern just shown and prove both **1.** and **2.** for a four-digit integer.