Completion Example 6 Factoring Trinomials

Completely factor each trinomial. Be sure to begin by looking for the greatest common factor.

a. $15x^2+38x+7$

b. $4y^2+6y-108$

Solution

a. $15x^2+38x+7=(5x+\_\_\_\_\_)(3x+\_\_\_\_\_)$

b. $4y^2+6y-108=2(\_\_\_\_\_y^2+\_\_\_\_\_y-\_\_\_\_\_)$

$=2(2y-\_\_\_\_\_)(y+\_\_\_\_\_)$

Now work margin exercise 6.

Completion Example Answers

2. a. $(5x+1)(3x+7)$ **b.** $2(2y^2+3y-54)=2(2y-9)(y+6)$

Margin Exercise Answers

1. a. $(x+6)(x+2)$ **b.** $(4u-7)(2u+3)$ **2. a.** $4x(2x-1)(x-1)$ **b.** $7x(3x^2+7x-1)$
3. $(3a+2)(a+4)$ **4.** $3(b-2)(4b+5)$ **5.** $(7x-2)(x+3)$ **6. a.** $(5x+3)(x-7)$
b. $3(x+4)(8x-3)$

6. Completely factor each trinomial. Be sure to begin by looking for the greatest common factor.

a. $5x^2-32x-21$

b. $24x^2+87x-36$

Note

No matter which method you use (the *ac*-method or the trial-and-error method), factoring trinomials takes time. With practice, you will become more efficient with either method. Make sure to be patient and observant.

13.3 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. When using the trial-and-error method to factor a trinomial of the form ax^2+bx+c, you first need to list all possible combinations of _____ of a and c, in their respective "First" and "Last" positions, according to the FOIL method.

2. The second step is to check the sums of the _____ in the O and I positions in the list until you find the sum to be c.

3. If none of these sums is c, the trinomial is not _____.

4. Look at the _____ term to determine what signs to use for the constants in the factors.

5. When using the *ac*-method of factoring, you need to find two integers whose _____ is ac and whose _____ is b.

6. The *ac*-method of factoring uses the _____ method.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. A trinomial is factorable if the middle term is the difference of the inner and outer products of two binomials.

8. The trial-and-error method of factoring a trinomial follows the same steps as the FOIL method of multiplication.

9. The first step in the *ac*-method of factoring is to rewrite the middle term.

10. Factoring can be checked by multiplying the factors and verifying that the product matches the original polynomial.

Practice

Completely factor each polynomial. If a polynomial cannot be factored, write "not factorable." See Examples 1 through 6.

1. x^2+5x+6

2. x^2-6x+8

3. $2x^2-3x-5$

4. $3x^2-4x-7$

5. $6x^2+11x+5$

6. $4x^2-11x+6$

7. $-x^2+3x-2$

8. $-x^2-5x-6$

9. $x^2-3x-10$

10. $x^2-11x+10$

11. $-x^2+13x+14$

12. $-x^2+12x-36$

13. $x^2+8x+64$

14. x^2+2x+3

15. $-2x^3+x^2+x$

16. $-2y^3-3y^2-y$

17. $4t^2-3t-1$

18. $2x^2-3x-2$

19. $5a^2-a-6$

20. $3a^2+4a+1$

21. $7x^2+5x-2$

22. $4x^2+23x+15$

23. $8x^2-10x-3$

24. $6x^2+23x+21$

25. $9x^2-3x-20$

26. $4x^2+40x+25$

27. $12x^2-38x+20$

28. $12b^2-12b+3$

29. $3x^2-7x+2$

30. $7x^2-11x-6$

31. $9x^2-6x+1$

32. $4x^2+4x+1$

33. $6y^2+7y+2$

34. $12y^2-7y-12$

35. $x^2-46x+45$

36. $x^2+6x-16$

37. $3x^2+9x+5$

38. $5a^2-7a+2$

39. $8a^2b-22ab+12b$

40. $12m^3n-50m^2n+8mn$

41. x^2+x+1

42. x^2+2x+2

43. $16x^2-8x+1$

44. $3x^2-11x-4$

45. $64x^2-48x+9$

46. $9x^2-12x+4$

47. $6x^2+2x-20$

48. $12y^2-15y+3$

49. $10x^2+35x+30$

50. $24y^2+4y-4$

51. $-18x^2+72x-8$

52. $7x^4-5x^3+3x^2$

53. $-45y^2+30y+120$

54. $-12m^2+22m+4$

55. $12x^2-60x+75$

56. $32y^2+50$

57. $6x^3+9x^2-6x$

58. $-5y^2+40y-60$

59. $12x^3-108x^2+243x$

60. $30a^3+51a^2+9a$

61. $9x^3y^3+9x^2y^3+9xy^3$

62. $48x^2y-354xy+126y$

63. $48xy^3-100xy^2+48xy$

64. $24a^2x^2+72a^2x+243x$

65. $21y^4-98y^3+56y^2$

66. $72a^3-306a^2+189a$

Writing & Thinking

67. It is true that $2x^2+10x+12=(2x+6)(x+2)=(2x+4)(x+3)$. Explain how the trinomial can be factored in two ways. Is there some kind of error?

68. It is true that $5x^2-5x-30=(5x-15)(x+2)$. Explain why this is not the completely factored form of the trinomial.

69. The volume of an open box is found by cutting equal squares (x inches on a side) from a sheet of cardboard that is 5 inches by 25 inches. The function representing this volume is $V(x)=4x^3-60x^2+125x$, where $0<x<2.5$. Factor this function and use the factors to explain, in your own words, how the function represents the volume.
(**Note:** Volume of a box = length × width × height.)