Solution

$$\begin{array}{r|rrrr} -3 & 3 & 10 & -5 & 0 & 125 \\ & & -9 & -3 & 24 & -72 \\ \hline & 3 & 1 & -8 & 24 & 53 \end{array} \leftarrow \text{Remainder} = P(-3)$$

Thus, $P(-3) = 53$.

Now work margin exercise 3.

4. Use synthetic division to show that $(x-2)$ is a factor of $P(x) = x^3 + x^2 + 2x - 16$.

Example 4 Using the Remainder Theorem and Synthetic Division

Use synthetic division to show that $(x-6)$ is a factor of $P(x) = x^3 - 14x^2 + 53x - 30$.

Solution

$$\begin{array}{r|rrrr} 6 & 1 & -14 & 53 & -30 \\ & & 6 & -48 & 30 \\ \hline & 1 & -8 & 5 & 0 \end{array} \leftarrow \text{Remainder} = P(6)$$

Thus, the remainder is $P(6) = 0$ and $\boldsymbol{(x-6)}$ **is a factor of** $\boldsymbol{P(x)}$.

Note: The coefficients in the quotient tell us that $x^2 - 8x + 5$ is also a factor of $P(x)$.

Now work margin exercise 4.

Margin Exercise Answers

1. a. $3x^2 - 2x + \dfrac{1}{x-1}$ **b.** $x^3 - x^2 + 3x - 4 + \dfrac{3}{x+3}$ **2.** $P(3) = 8$ 3. $P(-2) = 3$

4.
$$\begin{array}{r|rrrr} 2 & 1 & 1 & 2 & -16 \\ & & 2 & 6 & 16 \\ \hline & 1 & 3 & 8 & 0 \end{array}$$

12.9 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section

1. Synthetic division can be used to divide polynomials when the divisor of a rational expression is a first-degree _____ with leading coefficient 1.
2. Synthetic division involves omitting the _____ entirely and writing only certain coefficients.
3. If a polynomial $P(x)$ is divided by $(x-c)$ then the _____ will be $P(c)$.
4. When performing synthetic division, you first write only the _____ of the dividend and the _____ of the constant in the divisor.

5. Synthetic division results in a quotient that is a polynomial of _____ degree less than the dividend, along with the remainder.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

6. Synthetic division can be used to divide a polynomial by $2x + 3$.

7. At the end of the synthetic division process, the constants on the bottom line are the coefficients of the quotient and the remainder.

8. Synthetic division can be used to find the value of a polynomial for a particular value of x.

9. Synthetic division is only used when the divisor is a first-degree polynomial of the form $(x + c)$ or $(x - c)$.

Practice

Divide the following expressions using synthetic division. **a.** Write the answer in the form $Q + \frac{R}{D}$ where R is a constant. **b.** In each exercise, $D = (x - c)$. State the value of c and the value of $P(c)$. (Assume $P(x)$ is the numerator of the fraction.) See Examples 1 through 3.

1. $\dfrac{x^2 - 12x + 27}{x - 3}$

2. $\dfrac{x^2 - 12x + 35}{x - 5}$

3. $\dfrac{x^3 + 4x^2 + x - 1}{x + 8}$

4. $\dfrac{x^3 - 6x^2 + 8x - 5}{x - 2}$

5. $\dfrac{4x^3 + 2x^2 - 3x + 1}{x + 2}$

6. $\dfrac{3x^3 + 6x^2 + 8x - 5}{x + 1}$

7. $\dfrac{x^3 + 6x + 3}{x - 7}$

8. $\dfrac{2x^3 - 7x + 2}{x + 4}$

9. $\dfrac{2x^3 + 4x^2 - 9}{x + 3}$

10. $\dfrac{4x^3 - x^2 + 13}{x - 1}$

11. $\dfrac{x^4 - 3x^3 + 2x^2 - x + 2}{x - 3}$

12. $\dfrac{x^4 + x^3 - 4x^2 + x - 3}{x + 6}$

13. $\dfrac{x^4 + 2x^2 - 3x + 5}{x - 2}$

14. $\dfrac{3x^4 + 2x^3 + 2x^2 + x - 1}{x + 1}$

15. $\dfrac{x^4 - x^2 + 3}{x - \frac{1}{2}}$

16. $\dfrac{x^3 + 2x^2 + 1}{x - \frac{2}{3}}$

17. $\dfrac{x^5 - 1}{x - 1}$

18. $\dfrac{x^5 - x^3 + x}{x + \frac{1}{2}}$

19. $\dfrac{x^4 - 2x^3 + 4}{x + \frac{4}{5}}$

20. $\dfrac{x^6 + 1}{x + 1}$

Applications

Solve.

21. The minimum temperature for plants to grow in a certain model greenhouse can be modeled by the function $T(x) = 0.000027x^3 - 0.004144x^2 + 0.145x + 39.757$, where T is the degrees in Fahrenheit and x is the number of days since seeds were planted. Use synthetic division to find the minimum temperature after 15 days. Round your answer to the nearest tenth.

22. A moving company uses a box that has a volume of $x^3 + 7x^2 - 6x - 72$ cubic inches.

a. If the height of the box is $x + 4$, what is the area of the base of the box?

b. If the height of the box is $x - 3$, what is the area of the base of the box?

Collaborative Learning

23. With the class divided into teams of 3 or 4 students, each team should develop answers to the following questions and be prepared to discuss the answers in class.

a. First, use long division to divide the polynomial $P(x) = 2x^3 - 8x^2 + 10x + 15$ by $2x - 1$. Then, use synthetic division to divide the same polynomial by $x - \frac{1}{2}$. Do the same process with two or three other polynomials and divisors. Next compare the corresponding long and synthetic division answers and explain how the answers are related.

b. Use the results from part a. and explain algebraically the relationship of the answers when a polynomial is divided (using long division) by $ax - b$ and (using synthetic division) by $x - \frac{b}{a}$.

c. Show how the remainder theorem should be restated if $x - c$ is replaced by $ax - b$.