

A complete summary of the rules for exponents includes the following eight rules.

Summary of the Rules for Exponents

The following rules are true for any nonzero real numbers a and b and integers m and n .

1. The exponent 1: $a = a^1$
2. The exponent 0: $a^0 = 1$
3. The product rule: $a^m \cdot a^n = a^{m+n}$
4. The quotient rule: $\frac{a^m}{a^n} = a^{m-n}$
5. Negative exponents: $a^{-n} = \frac{1}{a^n}$.
6. Power rule: $(a^m)^n = a^{mn}$.
7. Power of a product: $(ab)^n = a^n b^n$.
8. Power of a quotient: $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$.

PROPERTIES

Margin Exercise Answers

1. a. x^{15} b. $\frac{1}{x^{12}}$ c. $\frac{1}{y^{15}}$ d. $\frac{1}{3^6}$ or $\frac{1}{9^3}$ or $\frac{1}{729}$ 2. a. $16x^2$ b. x^7y^7 c. $81a^2b^2$ d. $\frac{1}{a^3b^3}$
 e. $\frac{x^6}{y^8}$ 3. a. $\frac{x^7}{y^7}$ b. $\frac{25}{36}$ c. $\frac{27}{a^3}$ d. $\frac{x^3}{216}$ 4. a. $\frac{-27x^3}{y^9}$ b. $\frac{16b^2}{a^2}$ 5. $\frac{y^{15}}{x^{30}}$ 6. $\frac{64}{225x^{18}y^2}$

12.2 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. Moving any term from the numerator to the denominator, or vice versa, changes the sign of the corresponding _____.
2. A power of a quotient (in fraction form) is found by raising both the _____ and the _____ to that power.
3. To find the value of a power raised to a power, _____ the exponents and _____ the base.
4. A power of a product can be found by _____ each factor to that power.
5. In an expression such as $-x^2$, we know that -1 is understood to be the _____ of x^2 .

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

6. Taking the reciprocal of a fraction changes the sign of any exponent in the fraction.
7. For an exponent to refer to -7 as the base, -7 must be in parentheses.
8. When simplifying an expression with exponents, the rules for exponents must be used in a specific order or the answer will vary.
9. The expression -8^2 simplifies to -64 .

Practice

Use the rules for exponents to simplify each of the expressions. Assume that all variables represent nonzero real numbers. See Examples 1 through 6.

- | | | |
|---------------------|--------------------------------------|---|
| 1. -3^4 | 20. $(-3x^4)^2$ | 35. $\left(\frac{6m^3}{n^5}\right)^0$ |
| 2. -5^2 | 21. $4(-3x^2)^3$ | 36. $\left(\frac{3x^2}{y^3}\right)^2$ |
| 3. -2^4 | 22. $7(2y^{-2})^4$ | 37. $\left(\frac{-2x^2}{y^{-2}}\right)^2$ |
| 4. -20^2 | 23. $5(x^2y^{-1})$ | 38. $\left(\frac{2x}{y^5}\right)^{-2}$ |
| 5. $(-10)^6$ | 24. $-3(7xy^2)^0$ | 39. $\left(\frac{x}{y}\right)^{-2}$ |
| 6. $(-4)^6$ | 25. $-2(3x^5y^{-2})^{-3}$ | 40. $\left(\frac{2a}{b}\right)^{-1}$ |
| 7. $(a^3)^2$ | 26. $-4(5x^{-3}y)^{-1}$ | 41. $\left(\frac{3x}{y^{-2}}\right)^{-1}$ |
| 8. $(b^2)^{-4}$ | 27. $\left(\frac{a}{b}\right)^4$ | 42. $\left(\frac{4a^2}{b^{-3}}\right)^{-3}$ |
| 9. $(x^{-5})^2$ | 28. $\left(\frac{x}{2}\right)^3$ | 43. $\left(\frac{-3}{xy^2}\right)^{-3}$ |
| 10. $(x^{-2})^{-3}$ | 29. $\left(\frac{2}{3}\right)^2$ | 44. $\left(\frac{5xy^3}{y}\right)^2$ |
| 11. $(2^4)^{-2}$ | 30. $\left(\frac{a}{4}\right)^3$ | 45. $\left(\frac{m^2n^3}{mn}\right)^2$ |
| 12. $(2^{-3})^{-2}$ | 31. $\left(\frac{x}{y}\right)^6$ | |
| 13. $(3y)^2$ | 32. $\left(\frac{2}{5}\right)^2$ | |
| 14. $(ab)^4$ | 33. $\left(\frac{3x}{y}\right)^3$ | |
| 15. $(-4xy)^2$ | 34. $\left(\frac{-4x}{y^2}\right)^2$ | |
| 16. $(3x^{-2})^2$ | | |
| 17. $(xy)^{-6}$ | | |
| 18. $(a^3b^{-2})^3$ | | |
| 19. $(6x^3)^2$ | | |

46. $\left(\frac{2ab^3}{b^2}\right)^4$

47. $\left(\frac{-7^2x^2y}{y^3}\right)^{-1}$

48. $\left(\frac{2ab^4}{b^2}\right)^{-3}$

49. $\left(\frac{5x^3y}{y^2}\right)^2$

50. $\left(\frac{2x^2y}{y^3}\right)^{-4}$

51. $\left(\frac{x^3y^{-1}}{y^2}\right)^2$

52. $\left(\frac{2a^2b^{-1}}{b^2}\right)^3$

53. $\left(\frac{6y^5}{x^2y^{-2}}\right)^2$

54. $\left(\frac{3x^4}{x^{-2}y^{-4}}\right)^3$

55. $\frac{(7x^{-2}y)^2}{(xy^{-1})^2}$

56. $\frac{(-5x^3y^4)^2}{(3x^{-3}y)^2}$

57. $\frac{(3x^2y^{-1})^{-2}}{(6x^{-1}y)^{-3}}$

58. $\frac{(2x^{-3})^{-3}}{(5y^{-2})^{-2}}$

59. $\frac{(4x^{-2})(6x^5)}{(9y)(2y^{-1})}$


60. $\frac{(5x^2)(3x^{-1})^2}{(25y^3)(6y^{-2})}$

61. $\left(\frac{3xy^3}{4x^2y^{-3}}\right)^{-1}\left(\frac{2x^3y^{-1}}{9x^{-3}y^{-1}}\right)^2$

62. $\left(\frac{5a^4b^{-2}}{6a^{-4}b^3}\right)^{-2}\left(\frac{5a^3b^4}{2^{-2}a^{-2}b^{-2}}\right)^3$

63. $\left(\frac{6x^{-4}yz^{-2}}{4^{-1}x^{-4}y^3z^{-2}}\right)^{-1}\left(\frac{2^{-2}xyz^{-3}}{12x^2y^2z^{-1}}\right)^{-2}$

64. $\left(\frac{3^{-5}a^5b^3c^{-1}}{3^{-2}abc}\right)^{-2}\left(\frac{7^{-1}a^{-4}bc^2}{7^{-2}a^{-3}bc^{-2}}\right)^{-2}$

 Use a graphing calculator to evaluate each expression. Round quotients to the nearest ten-thousandth, if necessary.

65. $(2.1^2)^2$

66. $(1.4^{-2})^5$

67. $(3.8x)^4$

68. $(5.2x^2)^3$

69. $\left(\frac{8.1}{1.7}\right)^2$

70. $\left(\frac{2.3}{4.5}\right)^3$