Completion Example Answers

6. 199 is not even; $1 + 9 + 9 = 19$; $199 \div 7 = 28$ R3, $199 \div 11 = 18$ R1; $199 \div 13 = 15$ R4, $199 \div 17 = 11$ R12; 199 is a prime number

9. a. $6 \cdot 10 = 2 \cdot 3 \cdot 2 \cdot 5 = 2^2 \cdot 3 \cdot 5$ **b.** $4 \cdot 77 = 2 \cdot 2 \cdot 7 \cdot 11 = 2^2 \cdot 7 \cdot 11$

11. 1 and 154; 2, 7, and 11; $2 \cdot 7 = 14$, $2 \cdot 11 = 22$, $7 \cdot 11 = 77$; 1, 2, 7, 11, 14, 22, 77, and 154

Margin Exercise Answers

1. a. 13 has exactly two factors, 1 and 13. **b.** 19 has exactly two factors, 1 and 19. **2. a.** 1, 5, and 25 are all factors of 25. **b.** 1, 2, 4, 8, 16, and 32 are all factors of 32. **3.** No, the factors of 404 are 1, 2, 4, 101, 202, and 404, making 404 composite. **4.** No, the factors of 247 are 1, 13, 19, and 247, making 247 composite. **5.** Yes, tests for 2, 3, and 5 fail. Dividing by 7 and 11 fails to yield a divisor, and when dividing by 11, the quotient is less than 11. This means 113 is prime. **6.** Yes, tests for 2, 3, and 5 all fail. Dividing by 7, 11, 13, and 17 fails to yield a divisor, and when dividing by 17, the quotient is less than 17. This means 239 is prime. **7.** $2^2 \cdot 5 \cdot 7$ **8. a.** $2 \cdot 37$ **b.** $2^2 \cdot 5^2$ **c.** $2 \cdot 3^2 \cdot 13$ **9. a.** $2^2 \cdot 3 \cdot 23$ **b.** $2 \cdot 5^2 \cdot 11$ **10.** 1, 2, 3, 6, 7, 14, 21, and 42 **11.** 1, 3, 7, 9, 21, 63 **12.** 21 and 4

1.9 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. A composite number is a counting number with more than _____ different factors.

2. A prime number has exactly _____ different factors.

3. A prime number must be a number greater than _____.

4. Each composite number has exactly _____ prime factorization.

5. A prime number's factors are _____ and 1.

6. The number 1 is neither _____ nor _____.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. A prime number has exactly 1 factor.

8. A composite number has 2 or more factors.

9. 231 is a prime number.

10. All the factors of 30 are 1, 2, 3, 5, 6, 10, 15 and 30.

Practice

In your own words, write the definition of each term.

1. prime number

2. composite number

Determine whether each number is prime or composite. If the number is composite, find at least three factors of the number. See Examples 3 through 6, 10, and 11.

3. 47
4. 59
5. 63
6. 75
7. 103
8. 107
9. 205
10. 502
11. 143
12. 517

Two numbers are given. Find two factors of the first number such that their product is the first number and their sum is the second number. See Example 12.

13. 24, 10
14. 12, 7
15. 12, 13
16. 7, 8
17. 24, 11
18. 36, 15
19. 16, 8
20. 25, 10

Find the prime factorization of each number. Use the tests for divisibility for 2, 3, 4, 5, 6, 9, and 10 whenever they help to find beginning factors. See Examples 7 through 9.

21. 24
22. 28
23. 27
24. 125
25. 16
26. 81
27. 36
28. 72
29. 20
30. 50
31. 70
32. 105
33. 37
34. 43
35. 150
36. 120
37. 210
38. 330
39. 360
40. 600
41. 2200
42. 3500
43. 1000
44. 10,000

For each number, **a.** find the prime factorization and **b.** find all the factors.

45. 18
46. 28
47. 30
48. 42
49. 84
50. 90
51. 175
52. 275
53. 300
54. 700

Applications

Solve.

55. Twenty-four pencils are to be distributed evenly between the members of a group. What are the possible group sizes if each person in the group is to receive the same number of pencils?

56. Madeline plans to travel the world for a year and spend the same number of weeks in each country she visits. How many different countries can she visit during the year? (**Hint:** There are 52 weeks in a year.)

57. A chocolatier makes 72 specialty truffles. She wants to sell packages that each have the same number of truffles. What are her options for the number of truffles that can be in a package?

58. A radio station has 120 concert tickets to use to create prize packages to give to listeners. Each prize package will have the same number of tickets. What are the possible sizes for the packages of tickets?

Writing & Thinking

59. Are all odd numbers also prime numbers? Explain your answer.

60. Are all prime numbers also odd numbers? Explain your answer.

61. Explain how the tests for divisibility are helpful in finding prime factorizations.

62. Explain the difference between factors of a number and multiples of that number.

Collaborative Learning

63. In higher level mathematics, number theorists have proven the following theorem. Write the prime factorization of a number in exponential form. Add 1 to each exponent. The product of these sums is the number of factors of the original number.

For example, $60 = 2^2 \cdot 3 \cdot 5 = 2^2 \cdot 3^1 \cdot 5^1$. Adding 1 to each exponent and forming the product gives

$$(2+1)(1+1)(1+1) = 3 \cdot 2 \cdot 2 = 12$$

and there are twelve factors of 60.

Use this theorem to find the number of factors of each of the following numbers. Find as many of these factors as you can.

a. 700 **b.** 660 **c.** 450

64. Mathematicians have been interested since ancient times in searching for perfect numbers. A perfect number is a counting number that is equal to the sum of its proper divisors (divisors not including itself). For example, the first perfect number is 6. The proper divisors of 6 are 1, 2, and 3, and $1 + 2 + 3 = 6$. With the class separated into groups of 2 to 4 students, each team is to try to find the second and third perfect numbers. (**Hint:** The second perfect number is between 20 and 30, and the third perfect number is between 450 and 500.)