Objectives

A. Understand the basic concepts of fractions.

B. Graph fractions on a number line.

C. Understand the basic concepts of mixed numbers.

D. Graph mixed numbers on a number line.

E. Change mixed numbers to improper fractions.

F. Change improper fractions to mixed numbers.

2.1 Introduction to Fractions and Mixed Numbers

A Introduction to Fractions

Numbers such as $\frac{2}{3}$ (read "two-thirds") are said to be in **fraction form**. The top number, 2, is called the **numerator** and the bottom number, 3, is called the **denominator**.

$$\frac{2}{3} \quad \begin{matrix} \leftarrow \text{Numerator} \\ \leftarrow \text{Denominator} \end{matrix}$$

Fractions can be used to indicate parts of a whole. For example, if a whole candy bar has 7 equal parts, then the fraction $\frac{3}{7}$ (read "three-sevenths") indicates that we are considering 3 of those parts.

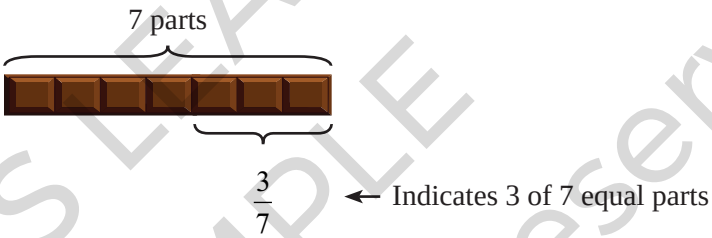


Figure 1

The whole candy bar can be represented as $\frac{7}{7}$.

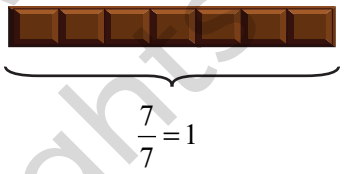


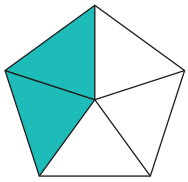
Figure 2

Example 1 shows several fractions indicating parts of a whole.

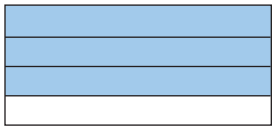
1. Write a fraction indicating

a. the shaded part of the figure and

b. the unshaded part of the figure.



Example 1 Understanding Fractions



Write a fraction indicating

a. the shaded part of the rectangle and

b. the unshaded part of the rectangle.

Solution

a. In the rectangle, 3 of the 4 equal parts are shaded. Thus, $\frac{3}{4}$ of the rectangle is shaded.

b. $\frac{1}{4}$ is not shaded.

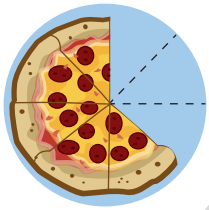
Now work margin exercise 1.

Example 2 Understanding Fractions

Write a fraction indicating

a. the remaining portion of the pizza and

b. the missing portion of the pizza.



Solution

a. The pizza was cut into 8 equal pieces. The 5 pieces remaining represent $\frac{5}{8}$ of the pizza.

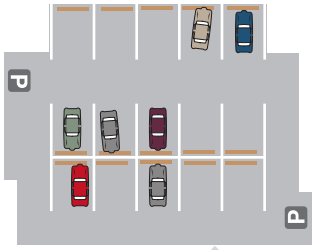
b. The missing portion of the pizza represents $\frac{3}{8}$ of the pizza.

Now work margin exercise 2.

2. Write a fraction indicating

a. the portion of the parking spaces that are occupied and

b. the portion of the parking spaces that are available.



Proper Fractions and Improper Fractions

A **proper fraction** is a fraction in which the numerator is less than the denominator. (Proper fractions have values less than 1.)

Examples of proper fractions: $\frac{2}{3}$, $\frac{7}{8}$, and $\frac{32}{60}$

An **improper fraction** is a fraction in which the numerator is greater than or equal to the denominator. (Improper fractions have values greater than or equal to 1.)

Examples of improper fractions: $\frac{15}{8}$, $\frac{14}{14}$, and $\frac{250}{100}$

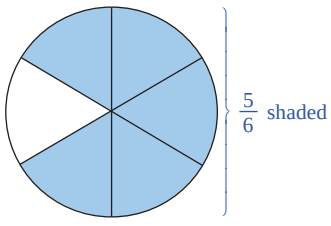
DEFINITION

Example 3 Understanding Proper Fractions

Draw a figure to represent the fraction $\frac{5}{6}$.

Solution

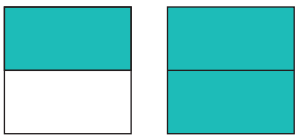
$\frac{5}{6}$ indicates 5 of 6 equal parts. Drawing a figure to represent this fraction, we divide a circle into 6 equal sections and shade 5 of them. (**Note:** Figures other than circles can be used.)



Now work margin exercise 3.

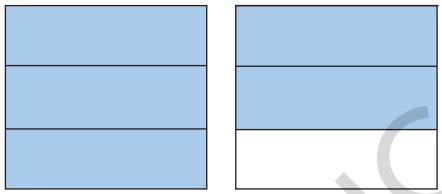
3. Draw a figure to represent the fraction $\frac{3}{8}$.

4. Write a fraction that indicates the shaded parts of the figure.



Example 4 Understanding Improper Fractions

Write a fraction that indicates the shaded parts of the figure.



Solution

There are two squares, each separated into 3 equal parts. This means that the denominator is 3. The shading here indicates 5 of these equal parts, which means the numerator is 5. Thus, the shaded part of the figure can be represented by the improper fraction $\frac{5}{3}$.

Now work margin exercise 4.

Whole numbers can be thought of as fractions with denominator 1. Thus, in fraction form

$$0=\frac{0}{1}, \quad 1=\frac{1}{1}, \quad 2=\frac{2}{1}, \quad 3=\frac{3}{1}, \quad \text{and so on.}$$

Fraction notation indicates division. For example, $24 \div 8$ can be written in the fraction form $\frac{24}{8}$, which indicates that the numerator is to be divided by the denominator. Thus,

$$\frac{24}{8}=3, \quad \frac{45}{5}=9, \quad \text{and} \quad \frac{0}{5}=0.$$

To discuss fractions in general we need the concept of a variable.

Variable

A **variable** is a symbol (generally a letter of the alphabet) that is used to represent an unknown number.

DEFINITION

Because we know that division by 0 is **undefined**, no denominator can be 0. Thus, in the fraction form $\frac{a}{b}$, we write $b \neq 0$ (read, "b is not equal to 0").

The Number 0 in Fractions

For any nonzero value of b, $\frac{0}{b}=\mathbf{0}$.

For any value of a, $\frac{a}{0}$ is **undefined**.

DEFINITION

Example 5 Evaluating Fractions Involving 0

Find the value of each expression.

a. $\frac{0}{36}$ **b.** $\frac{0}{124}$ **c.** $\frac{17}{0}$ **d.** $\frac{1}{0}$

Solution

a. $\frac{0}{36} = 0$ **b.** $\frac{0}{124} = 0$ **c.** $\frac{17}{0}$ is undefined **d.** $\frac{1}{0}$ is undefined

Now work margin exercise 5.

5. Find the value of each expression.

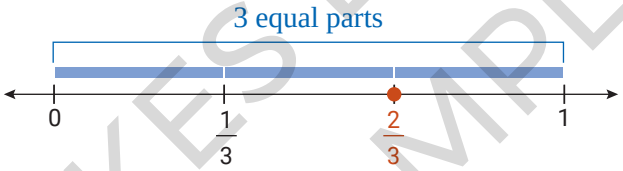
a. $\frac{0}{45}$

b. $\frac{10}{0}$

B Graphing Fractions on a Number Line

We have seen how to "picture" fractions as parts of a whole: part of a whole candy bar, part of a pizza, or part of a shaded region of a whole figure. Another way to visualize a fraction is to mark a corresponding point on a number line. For example, to graph the fraction $\frac{2}{3}$ proceed as follows.

1. Divide the interval (distance) from 0 to 1 into 3 equal parts.
2. Graph (or shade) the second mark to the right of 0.



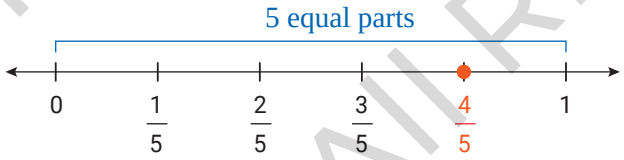
Example 6 Graphing Proper Fractions

Graph each proper fraction on a number line.

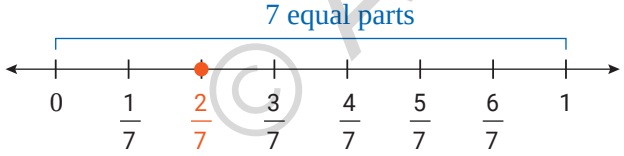
a. $\frac{4}{5}$ **b.** $\frac{2}{7}$ **c.** $\frac{5}{6}$

Solution

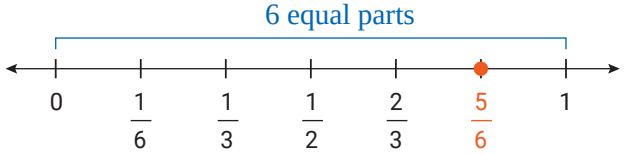
a.



b.



c.



Now work margin exercise 6.

6. Graph each proper fraction on a number line.

a. $\frac{3}{4}$

b. $\frac{1}{5}$

c. $\frac{1}{2}$

We know that with improper fractions the numerator is equal to or greater than the denominator. This means that the graph of an improper fraction will be at 1 or to the right of 1 on a number line.

7. Graph each improper fraction on a number line.

a. $\frac{4}{3}$

b. $\frac{9}{4}$

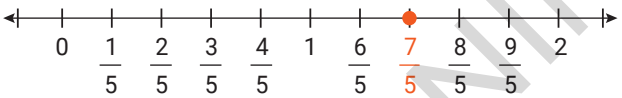
Example 7 Graphing Improper Fractions

Graph each of the following improper fractions on a number line.

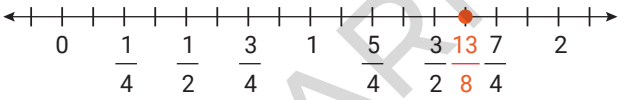
a. $\frac{7}{5}$ **b.** $\frac{13}{8}$

Solution

a.



b.



Now work margin exercise 7.

C Introduction to Mixed Numbers

A **mixed number** is the sum of a whole number and a proper fraction. By convention, we usually write the whole number and the fraction side by side without the plus sign. For example, $6+\frac{3}{5}=6\frac{3}{5}$ (read "six and three-fifths").

mixed number

$2\frac{1}{4}$ = whole number $\frac{\text{numerator}}{\text{denominator}}$

Terminology for mixed numbers

Typically, people are familiar with mixed numbers and use them frequently. For example, a carpenter might measure a board to be $2\frac{1}{4}$ feet long, or an architect might want to shade $2\frac{1}{4}$ circles in a drawing. A related question would be how many fourths (quarters) of a circle would be shaded? As shown in Figure 3, nine-fourths would be shaded. Thus, the form of an answer might be written as an improper fraction or a mixed number.

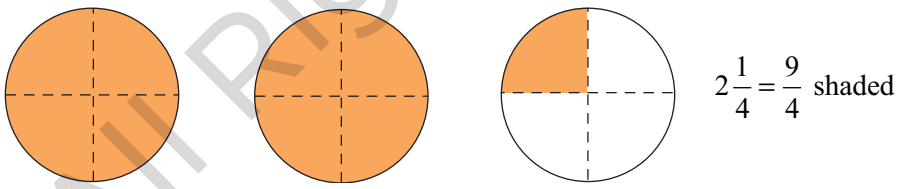


Figure 3

8. Identify each number as a proper fraction, an improper fraction, or a mixed number.

a. $5\frac{2}{3}$

b. $\frac{3}{2}$

c. $\frac{2}{3}$

Example 8 Identifying Types of Fractions and Mixed Numbers

Identify each number as a proper fraction, an improper fraction, or a mixed number.

a. $\frac{8}{7}$ **b.** $1\frac{6}{7}$ **c.** $\frac{7}{8}$

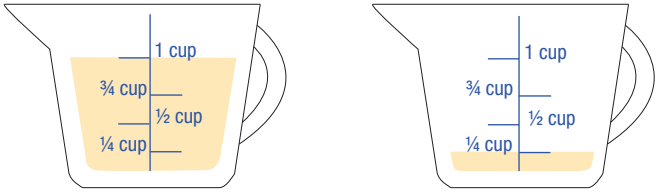
Solution

a. improper fraction **b.** mixed number **c.** proper fraction

Now work margin exercise 8.

Example 9 Application: Understanding Mixed Numbers

A recipe calls for the amount of oil indicated in the figure.



a. Write a mixed number indicating the amount of oil in the measuring cups.

b. Write this amount as an improper fraction.

Solution

a. Each cup is marked in fourths and we see that there is a total of $1\frac{1}{4}$ cups.

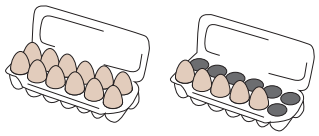
b. As an improper fraction, $1\frac{1}{4}$ cups $= \frac{5}{4}$ cups.

Now work margin exercise 9.

9. Cassandra has the following eggs in her refrigerator.

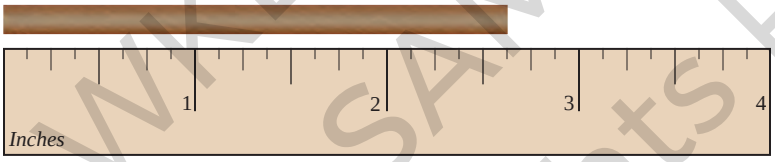
a. Write a mixed number indicating how many cartons of eggs she has (12 eggs equals 1 carton).

b. Write this amount as an improper fraction.



Example 10 Application: Understanding Mixed Numbers

A wooden rod is cut to the length indicated in the figure. Write the length of the rod as a mixed number.

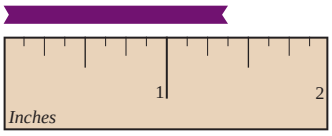


Solution

The ruler is marked in eighths of an inch. The rod measures $2\frac{5}{8}$ in.

Now work margin exercise 10.

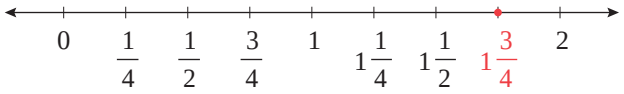
10. A ribbon is cut to the length indicated in the figure. Write the length of the ribbon as a mixed number.



D Graphing Mixed Numbers on a Number Line

We know that mixed numbers are greater than or equal to 1 with a whole number part and a fraction part. To graph a mixed number we can proceed as with fractions by making marks on a number line that correspond to the denominator of the fraction part. For example, to graph the mixed number $1\frac{3}{4}$ proceed as follows.

1. Mark the intervals from 0 to 1 and from 1 to 2 into 4 equal parts.
2. Graph (or shade) the mark at $1\frac{3}{4}$.



11. Graph each mixed number on a number line.

a. $1\frac{3}{4}$

b. $2\frac{2}{3}$

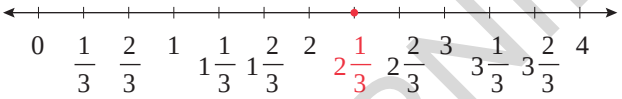
Example 11 Graphing Mixed Numbers

Graph each of the following mixed numbers on a number line.

a. $2\frac{1}{3}$ **b.** $3\frac{1}{2}$

Solution

a.



b.

Now work margin exercise 11.

E Changing Mixed Numbers to Improper Fractions

The following procedure can be used to change mixed numbers to improper fractions.

To Change a Mixed Number to an Improper Fraction

1. Multiply the whole number by the denominator of the proper fraction.
2. Add the numerator of the proper fraction to this product.
3. Write this sum over the denominator of the fraction.

1.	2.	3.
$2 \cdot 8 = 16$	$16 + 7 = 23$	
$2\frac{7}{8} =$	$2\frac{7}{8} =$	$\frac{23}{8}$

PROCEDURE

12. Change $10\frac{4}{9}$ to an improper fraction.

Example 12 Changing Mixed Numbers to Improper Fractions

Change $8\frac{9}{10}$ to an improper fraction.

Solution

Step 1: Multiply the whole number by the denominator: $8 \cdot 10 = 80$

Step 2: Add the numerator: $80 + 9 = 89$

Step 3: Write this sum over the denominator: $8\frac{9}{10} = \frac{89}{10}$

Now work margin exercise 12.

Completion Example 13 Mixed Numbers to Improper Fractions

Change $11\frac{2}{3}$ to an improper fraction.

Solution

Step 1: Multiply the whole number by the denominator: $11 \cdot 3 =$ _____

Step 2: Add the numerator: _____ + _____ = _____

Step 3: Write this sum over the denominator: $11\frac{2}{3} = \frac{\text{____}}{\text{____}}$

Now work margin exercise 13.

13. Change $3\frac{5}{9}$ to an improper fraction.

F Changing Improper Fractions to Mixed Numbers

To reverse the process (that is, to change an improper fraction to a mixed number), we use the fact that a fraction can indicate division.

To Change an Improper Fraction to a Mixed Number

1. Divide the numerator by the denominator. The quotient is the whole number part of the mixed number.
2. Write the remainder over the denominator as the fraction part of the mixed number.

PROCEDURE

Example 14 Changing Improper Fractions to Mixed Numbers

Change $\frac{67}{5}$ to a mixed number.

Solution

Step 1: Divide the numerator by the denominator. The quotient is the whole number part of the mixed number.

$$\begin{array}{r} 13 \\ 5\overline{)67} \\ \underline{-5} \\ 17 \\ \underline{-15} \\ 2 \end{array}$$

Whole number part

Remainder

Step 2: Write the remainder over the denominator as the fraction part of the mixed number:

14. Change $\frac{92}{11}$ to a mixed number.

$$\frac{67}{5} = 13 + \frac{2}{5} = 13\frac{2}{5}.$$

Now work margin exercise 14.

15. Change $\frac{77}{66}$ to a mixed number.

Example 15 Changing Improper Fractions to Mixed Numbers

Change $\frac{85}{2}$ to a mixed number.

Solution

Step 1: Divide the numerator by the denominator. The quotient is the whole number part of the mixed number.

$$\begin{array}{r} 42 \\ 2\overline{)85} \\ \underline{-8} \\ 05 \\ \underline{-4} \\ 1 \end{array}$$

Whole number part

Remainder

Step 2: Write the remainder over the denominator as the fraction part of the mixed number:

$$\frac{85}{2} = 42 + \frac{1}{2} = 42\frac{1}{2}.$$

Now work margin exercise 15.

CALCULATORS

Using a Calculator to Convert Between Fractions and Mixed Numbers

Many scientific calculators have a fraction button, $a\frac{b}{c}$. To enter a mixed number or fraction in your calculator using this button, press $a\frac{b}{c}$ between the whole number and the numerator, and again between the numerator and denominator. (To enter a fraction, simply press $a\frac{b}{c}$ between the numerator and denominator.)

If your calculator includes this feature, then you can use it to convert between improper fractions and mixed numbers. Consider the mixed number $2\frac{1}{3}$. To enter this mixed number in your calculator, press the keys

2 $a\frac{b}{c}$ 1 $a\frac{b}{c}$ 3.

The calculator will display this as 2_1_3, which means $2\frac{1}{3}$.

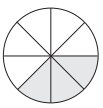
To convert this to an improper fraction press 2nd $a\frac{b}{c}$ (or SHIFT $a\frac{b}{c}$). This accesses the $A\frac{b}{c} \triangleleft\triangleright \frac{d}{e}$ feature that converts mixed numbers to fractions and vice versa. The display will read 7_3 which means $\frac{7}{3}$.

Similarly, to convert $\frac{5}{4}$ to a mixed number, press 5 $a\frac{b}{c}$ 4 =. The display will read 1_1_4 which means $1\frac{1}{4}$.

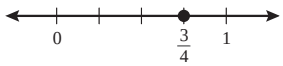
Completion Example Answers

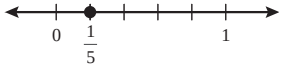
13. $11 \cdot 3 = 33$; $33 + 2 = 35$; $11\frac{2}{3} = \frac{35}{3}$

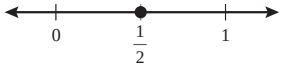
Margin Exercise Answers

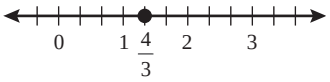
1. a. $\frac{2}{5}$ **b.** $\frac{3}{5}$ **2. a.** $\frac{7}{15}$ **b.** $\frac{8}{15}$ **3.**  **4.** $\frac{3}{2}$ (3 of the equal parts)

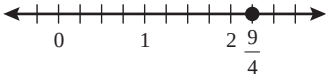
5. a. 0 **b.** Undefined

6. a. 

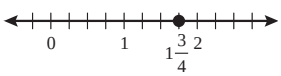
b. 

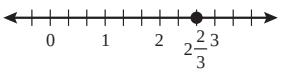
c. 

7. a. 

b. 

8. a. Mixed number **b.** Improper fraction **c.** Proper fraction **9. a.** $1\frac{5}{12}$ **b.** $\frac{17}{12}$ **10.** $1\frac{3}{8}$ in.

11. a. 

b. 

12. $\frac{94}{9}$ **13.** $\frac{32}{9}$ **14.** $8\frac{4}{11}$ **15.** $1\frac{11}{66}$

2.1 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

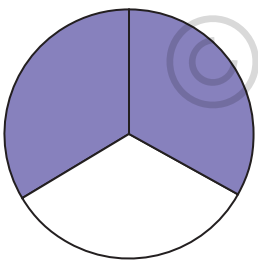
1. If a fraction has a numerator that is equal to or larger than the denominator, it is a/an _____ fraction.
2. A fraction that has a zero in the denominator is considered to be _____.
3. The sum of a whole number and a proper fraction is called a _____ number.
4. The first step in changing an improper fraction into a mixed number is to divide the _____ by the _____.

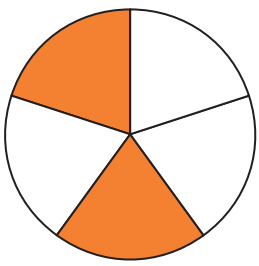
True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

5. In $\frac{11}{13}$, the denominator is 11.
6. $\frac{0}{6} = 0$
7. $\frac{17}{0}$ is undefined.

Practice

For each figure, write a fraction indicating **a.** the shaded part of the figure and **b.** the unshaded part of the figure. See Example 1.

1. 

2. 

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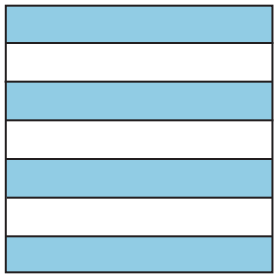

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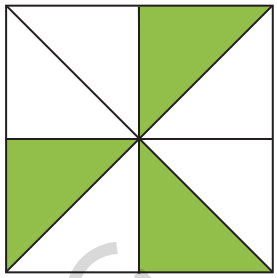
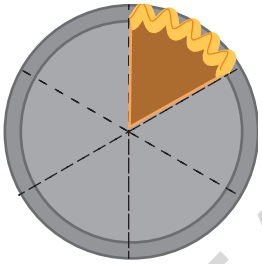


For each figure, write a fraction indicating **a.** the remaining portion of the object and **b.** the missing portion of the object. See Example 2.

5.



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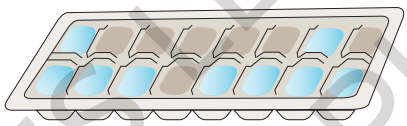


7.

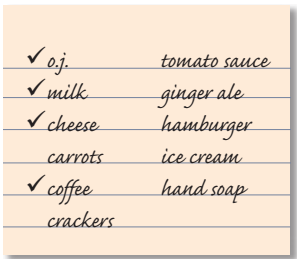
8.

9.

June

S	M	T	W	T	F	S
~~1~~	~~2~~	~~3~~	~~4~~	~~5~~	~~6~~	~~7~~
~~8~~	~~9~~	~~10~~	~~11~~	~~12~~	~~13~~	~~14~~
~~15~~	~~16~~	~~17~~	~~18~~	~~19~~	~~20~~	~~21~~
~~22~~	~~23~~	24	25	26	27	28
29	30					

10.



Draw a figure to represent each fraction. See Example 3.

11. $\frac{1}{3}$

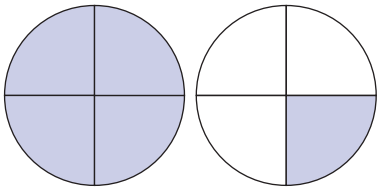
12. $\frac{1}{2}$

13. $\frac{4}{5}$

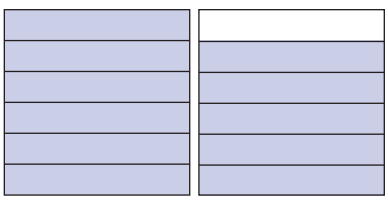
14. $\frac{3}{4}$

Write a fraction that indicates the shaded parts of each figure. See Example 4.

15.



16.



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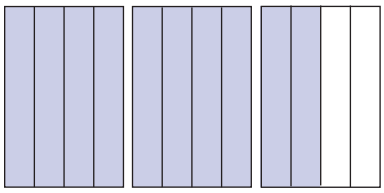
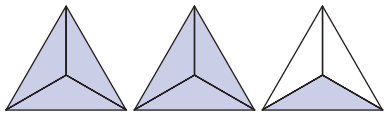


18.



Find the value of each expression. See Example 5.

19. $\frac{0}{6}$

20. $\frac{0}{35}$

21. $\frac{15}{0}$

22. $\frac{2}{0}$

Graph each fraction on a number line. See Examples 6 and 7.

23. $\frac{3}{5}$

24. $\frac{3}{8}$

25. $\frac{6}{5}$

26. $\frac{8}{3}$

Identify each number as a proper fraction, an improper fraction, or a mixed number. See Example 8.

27. $1\frac{1}{2}$

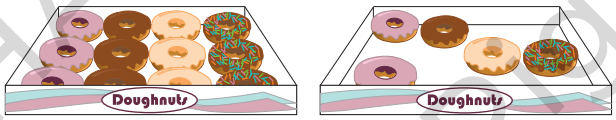
28. $\frac{5}{3}$

29. $\frac{7}{8}$

30. $7\frac{5}{12}$

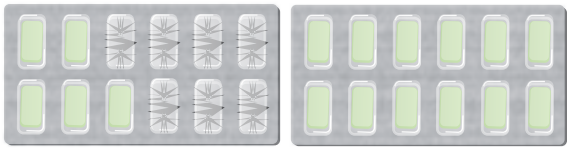
Write each amount described as **a.** a mixed number and **b.** an improper fraction. See Example 9.

31. Isabella brought 2 boxes of doughnuts to a meeting. The figure shows the remaining amount of doughnuts.

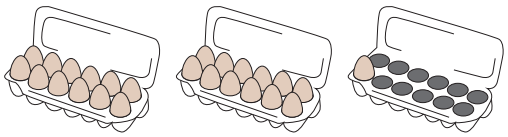


32. A recipe calls for the amount of tomato juice indicated in the figure.

33. Shane has two blister packs of gum. The figure shows the remaining amount of gum.

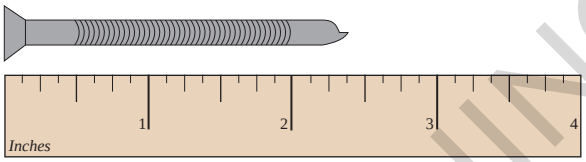


34. Cassandra has the following eggs in her refrigerator.

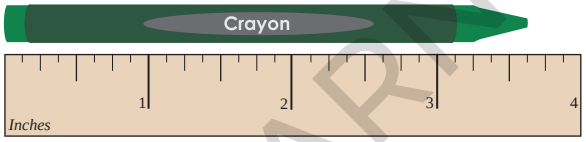


Write a mixed number to describe the length indicated in each figure. See Example 10.

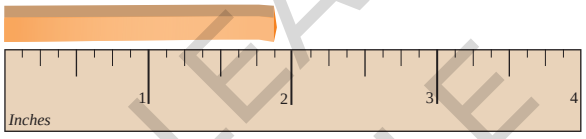
35.



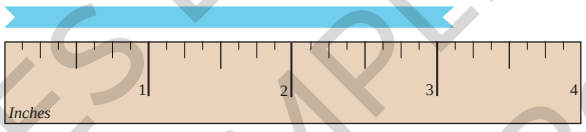
36.



37.



38.



Graph each mixed number on a number line. See Example 11.

39. $1\frac{1}{3}$ **40.** $3\frac{1}{4}$ **41.** $2\frac{2}{5}$ **42.** $1\frac{3}{8}$

Change each mixed number to an improper fraction. See Examples 12 and 13.

43. $1\frac{3}{5}$ **47.** $6\frac{8}{10}$ **51.** $4\frac{6}{7}$ **55.** $10\frac{8}{12}$

44. $1\frac{2}{3}$ **48.** $9\frac{4}{10}$ **52.** $7\frac{1}{7}$ **56.** $3\frac{1}{50}$

45. $2\frac{1}{4}$ **49.** $4\frac{5}{8}$ **53.** $1\frac{2}{15}$ **57.** $7\frac{1}{100}$

46. $4\frac{3}{4}$ **50.** $6\frac{4}{8}$ **54.** $1\frac{3}{17}$ **58.** $6\frac{19}{100}$

Change each improper fraction to a mixed number. See Examples 14 and 15.

59. $\frac{4}{3}$ **61.** $\frac{13}{2}$ **63.** $\frac{5}{2}$ **65.** $\frac{27}{10}$

60. $\frac{11}{8}$ **62.** $\frac{19}{4}$ **64.** $\frac{17}{8}$ **66.** $\frac{33}{10}$

67. $\frac{37}{8}$ **68.** $\frac{29}{6}$ **69.** $\frac{36}{12}$ **70.** $\frac{48}{16}$ **71.** $\frac{35}{20}$ **72.** $\frac{14}{5}$ **73.** $\frac{185}{100}$ **74.** $\frac{329}{100}$

Applications

Solve.

75. ***Eating Out:*** If you had $20 and you spent $9 for a hamburger, fries, and a soft drink, what fraction of your money did you spend? What fraction would you still have?

76. ***Grades:*** In a class of 35 students, 6 students received As on a mathematics exam. What fraction of students received an A? What fraction of students did not receive an A?

77. ***Customer Support:*** A software company receives 45 technical support calls in one hour. Twenty-three of the calls are related to customers forgetting their passwords. What fraction of the calls was related to customers forgetting their passwords?

78. ***Nutrition:*** A certain brand of plain bagels has 146 calories per bagel. 115 calories come from the carbohydrates in the bagel. What fraction of the calories is from carbohydrates?

79. ***Time:*** What fraction of a minute does 43 seconds represent? (**Hint:** There are 60 seconds in a minute.)

80. ***Distance:*** There are 5280 feet in a mile. What fraction of a mile does 923 feet represent?

81. ***Computers:*** A computer stores data on a hard drive in the form of bits, bytes, and sectors.

a. Each byte is made up of eight bits. What fraction of a byte is a bit?

b. A sector on a hard drive is traditionally 512 bytes. A byte is what fraction of a sector?

c. If a computer stores 159 bytes of data, what fraction of a sector does that amount of data take up?

82. ***Travel:*** The gas tank of a car holds 14 gallons of gas. What fraction of the tank does 9 gallons of gas take up?

83. ***Shipping:*** A small box will hold 12 books. Kathleen has 35 books to pack into small boxes.

a. Write an improper fraction to describe the number of boxes that will be filled by Kathleen's books.

b. Change the improper fraction from Part **a.** to a mixed number to describe the number of boxes that will be filled by Kathleen's books.

84. ***Beverages:*** A cup holds 8 ounces of liquid. You have 29 ounces of juice to pour into cups.

a. Write an improper fraction to describe the number of cups that will be filled with juice.

b. Change the improper fraction from Part **a.** to a mixed number to describe the number of cups that will be filled with juice.

Writing & Thinking

85. In your own words, list the parts of a fraction and briefly describe the purpose of each part.

86. Give an example of a situation where you might use fractions and/or mixed numbers outside of class.

87. Show and explain, using diagrams and words, why $2\frac{3}{5} = \frac{13}{5}$.

88. Explain how to change an improper fraction into a mixed number.