Example 9.4.1

Calculating a Confidence Interval for the Population Standard Deviation

The quality control supervisor of a bottling plant is concerned about the variance of fill per bottle. Regulatory agencies specify that the standard deviation of the amount of fill should be less than 0.1 ounce. To determine whether the process is meeting this specification, the supervisor randomly selects ten bottles, weighs the contents of each, and finds that the sample standard deviation of these measurements is 0.04. Assume that the data are collected from a normal population and compute a 95% confidence interval for the standard deviation of ounces of fill for the bottling plant.

SOLUTION

We want to find a 95% confidence interval for the variance. We are given that

$$n = 10,\ s = 0.04, \text{ and } \alpha = 0.05.$$

To calculate a 95% confidence interval, we use the following formula.

$$\frac{(n-1)s^2}{\chi^2_{\alpha/2}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{1-\alpha/2}}$$

Thus, we need to find the values of $\chi^2_{0.025}$ and $\chi^2_{0.975}$ for $n - 1 = 10 - 1 = 9$ degrees of freedom.

Using Table G in Appendix A, at 9 degrees of freedom,

$$\chi^2_{0.025} = 19.023$$
$$\chi^2_{0.975} = 2.700.$$

Substituting the values in the formula above, we have

$$\frac{(10-1)(0.04)^2}{19.023} < \sigma^2 < \frac{(10-1)(0.04)^2}{2.700}$$
$$0.000757 < \sigma^2 < 0.00533$$

So, a 95% confidence interval for the variance of fill of the bottles is between 0.000757 and 0.00533 ounce. However, the problem mentions the tolerance for the standard deviation of fill. So, to ensure that we make our interpretation in terms of the problem, to find a 95% confidence interval for the standard deviation, we take the square root of the confidence interval for the variance, yielding

$$0.0275 < \sigma < 0.0730.$$

The 95% confidence interval for the standard deviation of fill for the bottles is between 0.0275 and 0.0730 ounce, indicating that the process is meeting the specifications of being less than 0.1 ounce.

Technology

The confidence interval for the population variance can be obtained using Minitab. For detailed instructions, visit stat.hawkeslearning.com and navigate to **Discovering Business Statistics, Second Edition** > **Technology Instructions** > **Confidence Intervals** > **Variance**.

9.4 Exercises

Basic Concepts

1. What is the sampling distribution for $\frac{(n-1)s^2}{\sigma^2}$?
2. What assumption must hold to use the chi-square distribution to make inferences about the population variance?
3. True or false: the chi-square distribution is skewed to the right.
4. Give an example where we would want to calculate a confidence interval for σ^2

Exercises

5. A bolt manufacturer is very concerned about the consistency with which his machines produce bolts that are ¾ inch in diameter. When the manufacturing process is working normally the standard deviation of the bolt diameter is 0.05 inch. A random sample of 30 bolts has an average diameter of 0.25 inch with a standard deviation of 0.07 inch.

a. Construct a 95% confidence interval for the standard deviation of the bolt diameter. Interpret the interval.

b. What assumption did you make about the diameters of the bolts in constructing the confidence interval in part **a.**?

6. A drug that is used for treating cancer has potentially dangerous side effects if it is taken in doses that are larger than the required dosage for the treatment. The pharmaceutical company that manufactures the drug must be certain that the standard deviation of the drug content in the tablet is not more than 0.1 mg. Twenty-five tablets are randomly selected and the amount of drug in each tablet is measured. The sample has a mean of 20 mg and a variance of 0.015 mg.

a. Construct a 99% confidence interval for the variance of the amount of drug in each tablet. Interpret the interval.

b. What assumption did you make about the amounts of drug contained in the tablets in constructing the confidence interval in part **a.**?

7. A conservative investor would like to invest some money in a bond fund. The investor is concerned about the safety of her principal (the original money invested). Colonial Funds claims to have a bond fund which has maintained a consistent share price of $7. They claim that this share price has not varied by more than $0.25 on average since its inception. To test this claim, the investor randomly selects 25 days during the last year and determines the share price for the bond fund. The average share price of the sample is $7 with a standard deviation of $0.35.

a. Construct a 90% confidence interval for the standard deviation of the share price of the bond fund. Interpret the interval.

b. What assumption did you make about the share prices of the bond fund in constructing the confidence interval in part **a.**?

8. A manufacturer of automobile batteries is concerned about the life of the batteries that are produced. The manufacturer is comfortable with the average life of the batteries but more concerned about the standard deviation. Research has shown that the average life of the automobile batteries is 60 months. However, the manufacturer would like the standard deviation of the life of the automobile batteries to be relatively small, say, approximately six months. To determine a reliable range of the standard deviation of the batteries currently being produced, the manufacturer took a random sample of 15 batteries and found that the average life was 58 months with a standard deviation of seven months.

a. Construct a 98% confidence interval for the standard deviation of the life of their automobile batteries. Interpret this interval.

b. What assumptions did you make about the life of a battery being produced by the manufacturer?

9. Almost all smart devices (phones, tablets, and computers) are made with touch screens. A concern of many consumers is the shelf life of the "touch" component of the screens. A consumer advocacy group wanted to inform its members of a range that they can expect their touch screens to last. The group took a sample of 29 screens and measured the life of the "touch" function of the screens. That is, they used digital devices to simulate billions of touches to determine the life of the screens. Of the 29 screens sampled, the average "touch" life was 90 months with a standard deviation of six months. Construct an 80% confidence interval for the standard deviation of the life of the touch screens. Interpret this interval.

10. Photographers are always concerned about the number of shutter actuations that they will get from their cameras before they need to be serviced or the shutter needs to be replaced. To get an idea of the variability associated with the number of actuations, a photographer took a random sample of 20 cameras and found that the average number of actuations before failure was 200,000 with a standard deviation of 50,000.

a. Construct a 95% confidence interval for the standard deviation of the shutter actuations. Interpret the interval.

b. What assumptions did you make about the number of shutter actuations for the cameras?