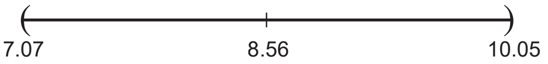
A 98% confidence interval is then calculated as follows.

$$\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

$$8.56 \pm 2.33\frac{7.85}{\sqrt{150}}$$

$$7.07 \text{ to } 10.05$$



Thus, we are 98% confident that the true mean number of new products introduced in the last 12 months will be contained in the above interval.

Technology

For instructions on calculating this confidence interval using technology, please visit stat.hawkeslearning.com and navigate to **Discovering Business Statistics, Second Edition > Technology Instructions > Confidence Intervals >** ***z*-Interval**.

So far, the confidence interval has been discussed as a way of placing bounds on the location of a parameter with a specific degree of confidence. But we can also think about the confidence interval as a means of describing the quality of a point estimate. Let's look at the expression for the confidence interval for the population mean.

Confidence Interval for μ

$$\underbrace{\bar{x}}_{\text{point estimate}} \pm \underbrace{z_{\alpha/2}\frac{\sigma}{\sqrt{n}}}_{\text{margin of error with a specific level of confidence}}$$

Another interpretation of the confidence interval is given below the expression of the confidence interval for μ. The part of the expression that is added and subtracted to the point estimate, $z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$, can be thought of as the **margin of error** (also known as the **maximum error of estimation**) using the point estimate $\bar{x}$ with a specified level of confidence. For example, the 95% confidence interval in Example 9.1.1 was given as

$$425 \pm 1.96 \cdot \frac{900}{\sqrt{100}}$$

$$425 \pm 176.4.$$

We could say that we are 95% confident that the point estimate of μ, $\bar{x} = 425$, has a margin of error of 176.4 or an error of estimation no larger than 176.4. Being able to assess the error of an estimate is one of the most useful applications of statistical methods.

Definition

Margin of Error

The **margin of error**, or **maximum error of estimation** (often denoted as E), is the largest possible distance from the point estimate that a confidence interval will cover.

9.1 Exercises

Basic Concepts

1. What is statistical inference?
2. What is an estimator?
3. What is a judgment estimate? What are some drawbacks of judgment estimates?
4. Explain, in your own words, the difference between the terms *estimator* and *estimate*.
5. What is the difference between a point estimate and an interval estimate?
6. Give three examples of point estimators. Identify the parameters being estimated by these estimators.
7. Describe the primary advantages of *random* sampling procedures.
8. What are two important questions to consider when estimating a population mean?

9. What is mean squared error?

10. What is an unbiased estimator? Give an example.

11. Why is the sample mean considered the best point estimate of the population mean?

12. Are all estimators unbiased? Explain.

13. Generally, we expect most sample statistics to be good estimators of their population counterparts. Which statistic is the exception to this idea?

14. What are two characteristics of the best available estimate for a parameter?

15. What is an interval estimator?

16. What is the distinction between probability and confidence?

17. What is the role of the z-value in the confidence interval expression?

18. Describe in words the ideas behind the construction of a confidence interval.

19. Consider the following statement: *If the sample size is greater than or equal to 30, then by the Central Limit Theorem there is a 0.95 probability that the sample mean will be within 1.96 standard deviations of the population mean before a particular sample is selected.* Explain why the phrase "before a particular sample is selected" is important here.

20. Explain what is wrong with the following expression: $P(111 < \mu < 189) = 0.95$.

21. Define the following terms: confidence level, confidence coefficient, confidence interval.

22. What are the conditions required in order to construct a $100(1-\alpha)\%$ confidence interval using the expression $\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$?

23. Describe the effect on the width of a confidence interval as each of the following increases:

a. n **b.** $1-\alpha$ **c.** α **d.** $\bar{x}$

24. What expression indicates the margin of error? Is this the same as the maximum error of estimation?

Exercises

25. Find $z_{\alpha/2}$ for the following levels of α.

a. $\alpha = 0.05$ **b.** $\alpha = 0.01$ **c.** $\alpha = 0.10$

26. Find $z_{\alpha/2}$ for the following levels of α.

a. $\alpha = 0.04$ **b.** $\alpha = 0.02$ **c.** $\alpha = 0.08$

27. Find $z_{\alpha/2}$ for the following confidence levels.

a. 98% **b.** 94% **c.** 92%

28. Find $z_{\alpha/2}$ for the following confidence levels.

a. 96% **b.** 88% **c.** 85%

29. Consider a normally distributed population with a standard deviation of 64. If a random sample of size 90 from the population produces a sample mean of 250, construct a 95% confidence interval for the true mean of the population.

30. Construct a 90% confidence interval for the true mean of a normal population if a random sample of size 40 from the population yields a sample mean of 75 and the population has a standard deviation of 5.

31. A psychologist is studying learning in rats. The psychologist wants to determine the average time required for rats to learn to traverse a maze. She randomly selects 40 rats and records the time it takes for the rats to traverse the maze in minutes. The sample average time required for the rats to traverse the maze is 5 minutes with a population standard deviation of 1 minute. Estimate the average time required for rats to learn to traverse the maze with a 90% confidence interval.

32. A paint manufacturer is developing a new type of paint. Thirty panels were exposed to various corrosive conditions to measure the protective ability of the paint. The mean life for the samples was 168 hours before corrosive failure. The life of paint samples is assumed to be normally distributed with a population standard deviation of 30 hours. Find the 95% confidence interval for the mean life of the paint.

33. The chief purchaser for the State Education Commission is reviewing test data for a metal link chain which will be used on children's swing sets in elementary school playgrounds. The average breaking strength for a sample of 50 pieces of chain is 5000 pounds. Based on past experience, the breaking strength of metal chains is known to be normally distributed with a standard deviation of 100 pounds. Estimate the actual mean breaking strength of the metal link chain with 99% confidence.

34. Tomatoes are grown in Florida for shipment to other parts of the country by the Anderson Produce Company. A random sample of 40 boxes is selected at one warehouse for weighing. The average weight for the sample is 33.5 pounds per box with a population standard deviation of 2.1 pounds. Find a 90% confidence interval for the true average weight of the boxes of tomatoes.

35. Thirty-five strands of piano wire were selected at random from a recent shipment by the quality control department at Elkins Piano Company. The strands of piano wire were tested to failure in tests of tensile strength. The mean tensile strength of the sample was 30,000 pounds per square inch (psi) with a population standard deviation of 1950 psi. Find the 98% confidence interval for the true mean tensile strength.

36. When preparing a standardized test to be given to all the sixth graders, the Standard Test Company gave a version of the test to a random sample of 45 sixth graders and timed how long it took them to finish the test. The average time required to finish the test for the sample was 2 hours and 15 minutes with a population standard deviation of 30 minutes. Estimate the true average time required to finish the test with 95% confidence.

37. According to the 2009 College Senior Survey administered by the Higher Education Research Institute at UCLA, 56.4% of college seniors spend 10 hours or less studying or doing homework in a typical week. Suppose a random sample of 50 college seniors was selected from all the college seniors in the Southeast region to determine the homework habits of college seniors in the Southeast region of the United States. Each student in the sample is asked approximately how many hours per week he or she spends studying or doing homework. If the mean is 9.6 hours and the population standard deviation is 3.1 hours, construct a 99% confidence interval for the mean number of hours a week that a college senior in the region spends studying or doing homework per week.

Source: Cooperative Institutional Research Program at the Higher Education Research Institute at UCLA

9.2 Estimating the Population Mean, σ Unknown

In the previous section, we assumed that the population standard deviation was known. In practice this assumption is not very realistic, since the standard deviation describes variability about the mean. If the population standard deviation is known, the mean is usually also known, and there is no need to create an interval estimate for it. Why estimate something we already know?