

($np \geq 5$ and $n(1-p) \geq 5$), the distribution of \hat{p} will be approximately normal with

$$\begin{aligned}\mu_{\hat{p}} &= p = 0.6 \\ \sigma_{\hat{p}} &= \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.6(1-0.6)}{150}} = 0.04.\end{aligned}$$

To find the probability that \hat{p} is within 0.05 of the true proportion, we must find

$$P(p - 0.05 < \hat{p} < p + 0.05) = P(0.55 < \hat{p} < 0.65).$$

Sampling Distribution of \hat{p}

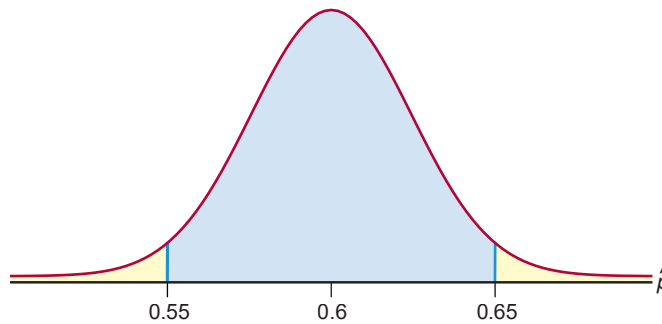


Figure 8.3.6

Using the z -transformation,

$$\begin{aligned}&= P\left(\frac{0.55 - 0.6}{0.04} < z < \frac{0.65 - 0.6}{0.04}\right) \\ &= P(-1.25 < z < 1.25) \\ &= P(z < 1.25) - P(z < -1.25) \\ &= 0.8944 - 0.1056 \\ &= 0.7888.\end{aligned}$$

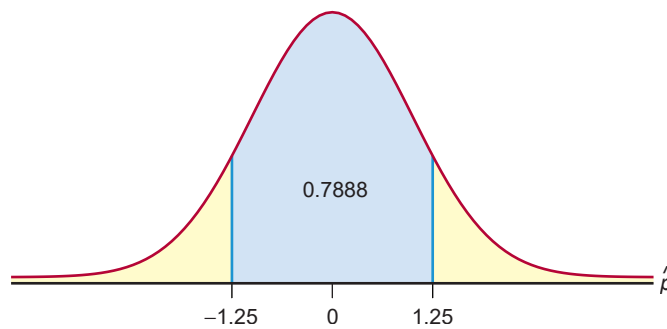


Figure 8.3.7

Therefore, for a sample of 150 U.S. citizens over 18 years of age, it is likely (0.7888) that the error of estimation will be less than five percentage points.

Technology

For instructions on calculating this probability using technology, please visit stat.hawkeslearning.com and navigate to **Discovering Business Statistics, Second Edition > Technology Instructions > Normal Distributions > Normal Probability (cdf)**.

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NORMAL FLOAT AUTO REAL RADIAN MP
normalcdf(0.55,0.65,0.6,0)
.....0.7887003221
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8.3 Exercises

Basic Concepts

1. What does the symbol \hat{p} represent?
2. What is the connection between \hat{p} and p ?

3. Is \hat{p} an unbiased estimator? If so, of what?
4. What are the conditions that make the sample size n “sufficiently large” for a sample proportion?
5. Describe the sampling distribution of \hat{p} if n is sufficiently large.

Exercises

6. A random sample of 40 electronic components has 5 defective components.
 - a. Find the sample proportion of components that are defective.
 - b. Find the sample proportion of components that are not defective.
7. A random sample of 100 employees of a large steel company has 30 females and 70 males.
 - a. Find the sample proportion of female employees.
 - b. Find the sample proportion of male employees.
8. Suppose that the true proportion of registered voters who favor the Republican presidential candidate is 0.45. Find the mean and standard deviation of the sample proportion for samples of the following sizes.
 - a. $n = 30$
 - b. $n = 45$
 - c. $n = 65$
 - d. What happens to the size of the standard deviation of the sample proportion as the sample size increases?
9. Suppose that the true proportion of Americans over 25 years old that have a 4-year college degree is 0.35. Find the mean and the standard deviation of the sample proportion for samples of the following sizes.
 - a. $n = 38$
 - b. $n = 52$
 - c. $n = 75$
 - d. What happens to the size of the standard deviation of the sample proportion as the sample size increases?
10. Suppose the true population proportion is $p = 0.50$. What is the probability that the sample proportion of a sample of size 20 will be greater than 0.60?
11. Suppose the true population proportion is $p = 0.30$. What is the probability that the sample proportion of a sample of size 30 will be less than 0.20?
12. Suppose that the true proportion of Americans who save at least 10% of their income is 0.15. If \hat{p} is the sample proportion of Americans surveyed who save at least 10% of their income from a sample of size 38, find the following probabilities.

a. $P(\hat{p} > 0.25)$	c. $P(0.10 < \hat{p} < 0.20)$
b. $P(\hat{p} < 0.09)$	d. $P(0.18 < \hat{p} < 0.25)$
13. Suppose that the true proportion of airline pilots between the ages of 35 and 45 is 0.60. If \hat{p} is the sample proportion of airline pilots between the ages of 35 and 45 from a sample of size 100, find the following probabilities.

a. $P(\hat{p} > 0.55)$	c. $P(0.50 < \hat{p} < 0.60)$
b. $P(\hat{p} < 0.45)$	d. $P(0.60 < \hat{p} < 0.75)$

14. The director of a radio station in a large metropolitan area believes that the proportion of young professionals (his target market) in the area who prefer rock 'n' roll music has increased from 25% to 35%. The director randomly decides to select 50 young professionals and ask them if they prefer rock 'n' roll to any other type of music. If the sample proportion is greater than 0.35, he will switch to a new format emphasizing rock 'n' roll.
 - a. If the true proportion of young professionals who prefer rock 'n' roll has not changed, find the probability that the radio director will switch to the new format.
 - b. If the true proportion of young professionals who prefer rock 'n' roll has changed as the director suspects, find the probability that the radio director will switch to the new format.
15. The property manager of a large office building would like to make the building smoke free; however, he does not want to upset too many of his customers. He decides to randomly select 50 of the workers in the building and ask them whether or not they smoke. If the sample proportion of workers who smoke is less than 0.30, the property manager will make the building smoke free.
 - a. Find the probability that the property manager will make the building smoke free when the true proportion of smokers is 0.5.
 - b. Find the probability that the property manager will not make the building smoke free when the true proportion of smokers is 0.2.
16. Eighty percent of the flights arriving in Atlanta for a large U.S. airline are on time. If the FAA randomly selects 50 of the airline's flights, find the probability that:
 - a. at least 85% of the sampled flights will be on time.
 - b. at most 70% of the sampled flights will be on time.
 - c. between 75% and 85% of the sampled flights will be on time.
17. Approximately 7% of the nation's public school children in grades 2 through 5 took medication in 2003 for attention deficit hyperactivity disorder (ADHD), a developmental disorder characterized by impulsiveness or difficulty concentrating or sitting still. The main treatment prescribed for ADHD is Ritalin, a relatively safe drug with few side effects. Assume that a suburban elementary school had an enrollment of 286 students in 2003.
 - a. Find the probability that at least 4% of the school children took medication for ADHD.
 - b. Find the probability that between 5% and 8% of the school children took medication for ADHD.

8.4 Sampling Methods

Random sampling is an effective means of obtaining a sample that is representative of the population. As we discussed previously, acquiring an exact sampling frame for the population under consideration is a requirement for simple random sampling, a requirement which can be time-consuming and expensive. In addition, it is sometimes not even possible to list all the members of a population. There are other sampling strategies that are designed to reduce the cost of sampling or add control to the sampling procedure. These techniques can be categorized as probability samples or non-probability samples.

Probability samples enable an analyst to determine the probable errors that an estimator might generate. Essentially, they allow the analyst a known degree of confidence in his or her estimation. All of statistical inference relies on probability sampling. **Non-probability samples**

Definition

Probability Sample

A **probability sample** is a sample used to estimate a population parameter that has known errors, allowing a statement about the reliability of the estimate to be made.