8.2 Exercises

Basic Concepts

1. What key three questions should be asked when considering a random variable?

2. Explain the difference between a biased estimator and an unbiased estimator.

3. Give three examples of estimators that are unbiased.

4. Is an unbiased estimator always closer to the parameter being estimated than a biased estimator? Explain.

5. What is the standard error of the mean? What does it indicate?

6. What are two desirable characteristics of the sample mean?

7. Explain the Central Limit Theorem.

8. What effect does increasing the sample size have on the accuracy of an estimate?

9. What is the error of estimation?

Exercises

10. Suppose the random variable X has a mean of 20 and a standard deviation of 5. Calculate the mean and the standard deviation of the sample mean for each of the following sample sizes (assume the population is infinite).

a. $n = 35$

b. $n = 50$

c. $n = 75$

d. What happens to the size of the standard deviation of the sample mean as the sample size increases?

11. Suppose the random variable X has a mean of 50 and a standard deviation of 10. Calculate the mean and standard error for each of the following sample sizes (assume the population is infinite).

a. $n = 40$

b. $n = 55$

c. $n = 100$

d. What happens to the size of the standard error as the sample size increases?

12. If there is a normally distributed random variable with a mean of 75 and a standard deviation of 22, what is the probability that the mean of a sample of size 19 will be greater than 80?

13. If a sample of size 40 is drawn from a population that has a mean of 276 and a variance of 81, what is the probability that the mean of the sample will be less than 273?

14. Suppose there is a normally distributed population with a mean of 250 and a standard deviation of 50. If $\bar{x}$ is the average of a sample of 36, find the following probabilities.

a. $P(\bar{x} \le 240)$

b. $P(\bar{x} \ge 255)$

c. $P(246 \le \bar{x} \le 260)$

d. $P(234 \le \bar{x} \le 245)$

15. Suppose there is a normally distributed population with a mean of 100 and a standard deviation of 10. If $\bar{x}$ is the average of a sample of 50, find the following probabilities.

a. $P(\bar{x} \le 110)$

b. $P(\bar{x} \ge 90)$

c. $P(95 \le \bar{x} \le 115)$

d. $P(85 \le \bar{x} \le 98)$

16. A company fills bags with fertilizer for retail sale. The weights of the bags of fertilizer have a normal distribution with a mean weight of 15 lb and standard deviation of 1.70 lb.

- **a.** What is the probability that a randomly selected bag of fertilizer will weigh between 14 and 16 pounds?
- **b.** If 35 bags of fertilizer are randomly selected, find the probability that the average weight of the 35 bags will be between 14 and 16 pounds.

17. A travel agency conducted a survey of the prices charged by ocean cruise ship lines and determined they were approximately normally distributed with a mean of $110 per day and a standard deviation of $20 per day.

- **a.** If an ocean cruise ship line is chosen at random, find the probability that it will charge less than $99 per day.
- **b.** What is the probability that the average charge for a randomly selected sample of 35 ocean cruise ship lines will be less than $99 per day?

18. The turkeys found in a particular county have an average weight of 15.6 pounds with a standard deviation of 4.00 pounds. Forty-five turkeys are randomly selected for a county fair.

- **a.** Find the probability that the average weight of the turkeys will be less than 14.5 pounds.
- **b.** What is the probability that the average weight of the turkeys will be more than 17 pounds?
- **c.** Find the probability that the average weight of the turkeys will be between 13 and 18 pounds.

19. The average score for a water safety instructor (WSI) exam is 75 with a standard deviation of 12. Fifty scores for the WSI exam are randomly selected.

- **a.** Find the probability that the average of the fifty scores is at least 80.
- **b.** Find the probability that the average of the fifty scores is at most 70.
- **c.** Find the probability that the average of the fifty scores is between 72 and 78.

20. A college food service buys frozen fish in boxes labeled 10 pounds. The true average weight of the boxes is 8 pounds with a standard deviation of 2 pounds. The food service director suspects that the boxes do not contain as much fish as advertised. He decides to inspect 40 boxes from the next shipment. If the average weight is less than 10 pounds he will reject the entire shipment. Find the probability that the food service director will not reject the shipment.

21. The AQI, or the Air Quality Index, is an index used to determine the ozone level in a city. Depending upon the AQI reading, it may not be safe to jog or even to go outside. Readings in the 0–50 range mean that the air quality conditions are considered "good," 51–100 are "moderate," 101–150 means "unhealthy for sensitive groups," 151–200 means "unhealthy," 201–300 means "very unhealthy," and 301–500 means "hazardous." Suppose that an industrial region has an average AQI reading of 102 with a standard deviation of 40. Find the probability that for a random sample of 50 days, the average AQI reading is:

Source: airnow.gov

- **a.** at least 105.
- **b.** at most 90.
- **c.** between 100 and 115.