Example 6.4.4

Calculating the Expected Value and Variance of a Binomial Random Variable

Compute the expected value and the variance of the number of customers that will approve the change in return policy in Example 6.4.3.

SOLUTION

Since the random variable is binomial, we can use the shortcuts $E(X) = np$ and $V(X) = np(1 - p)$. Since $n = 10$ and $p = 0.7$, the expected value is given by the following expression.

$$E(X) = np = 10(0.7) = 7$$

The variance is

$$\sigma^2 = V(X) = np(1 - p) = 10(0.7)(0.3) = 2.1,$$

which implies that the standard deviation is $\sqrt{2.1} \approx 1.4491$.

Thus, if 10 randomly selected customers are polled, we would expect 7 of the 10 to approve the change in return policy, and the standard deviation would be 1.4491 customers.

6.4 Exercises

Basic Concepts

1. Describe the characteristics of a binomial experiment.
2. What are the parameters of a binomial probability model?
3. Give an example of a binomial experiment in a business context.
4. What is the binomial probability distribution function?
5. Describe the shape of a binomial distribution. Does the shape change? What influences the shape of the distribution?
6. How do you calculate the expected value of a binomial random variable? The variance? The standard deviation?

Exercises

7. Calculate ${}_nC_x$ for each of the following combinations of x and n.
   - **a.** $n = 5, x = 4$
   - **b.** $n = 10, x = 8$
   - **c.** $n = 15, x = 1$
   - **d.** $n = 20, x = 0$
8. Calculate ${}_nC_x$ for each of the following combinations of x and n.
   - **a.** $n = 4, x = 2$
   - **b.** $n = 12, x = 8$
   - **c.** $n = 18, x = 15$
   - **d.** $n = 23, x = 20$
9. The random variable X is a binomial random variable with n = 9 and p = 0.1.
   - **a.** Find the expected value of X.
   - **b.** Find the standard deviation of X.
   - **c.** Find the probability that X equals 2. (Use the formula for $P(X = x)$.)
   - **d.** Find the probability that X is at most 3.
   - **e.** Find the probability that X is at least 2.
   - **f.** Find the probability that X is less than 5.

10. The random variable X is a binomial random variable with $n = 12$ and $p = 0.8$.

a. Find the expected value of X.

b. Find the standard deviation of X.

c. Find the probability that X equals 7. (Use the formula for $P(X = x)$.)

d. Find the probability that X is at most 4.

e. Find the probability that X is at least 1.

f. Find the probability that X is more than 10.

11. A real estate agent has ten properties that she shows. She feels that there is a ten percent chance of selling any one property during a week. The chance of selling any one property is independent of selling another property.

a. What probability model would be appropriate for describing the number of properties sold each week?

b. Compute the expected number of properties to be sold in a week.

c. Compute the standard deviation of the number of properties sold each week.

d. Compute the probability of selling one property in one week.

e. Compute the probability of selling five properties in one week.

f. Compute the probability of selling at least three properties in one week.

12. A small commuter airline is concerned about reservation no-shows and, correspondingly, how much they should overbook flights to compensate. Assume their commuter planes will hold 15 people. Industry research indicates that 20% of the people making a reservation will not show up for a flight. Whether or not one person takes the flight is considered to be independent of other persons holding reservations.

a. What probability model would be appropriate for the number of passengers that actually take the flight?

b. If the airlines decide to book 18 people for each flight, how often will there be at least one person who will not get a seat?

c. If they book 17 people, how often will there be at least one person who will not get a seat?

d. If they book 16 people, how often will there be at least one person who will not get a seat?

e. If they book 18 people for each flight, how often will there be one or more empty seats?

f. If they book 17 people, how often will there be one or more empty seats?

g. If they book 16 people, how often will there be one or more empty seats?

h. Based on the results from parts **b.** to **g.** above, which booking policy do you prefer? Explain your answer.

13. Seven plants are operated by a garment manufacturer. They feel there is a ten percent chance for a strike at any one plant and the risk of a strike at one plant is independent of the risk of a strike at another plant. Let X = number of plants of the garment manufacturer that strike.

a. Determine the probability distribution for X.

b. Interpret the results for $P(X = 0)$, $P(X = 4)$, and $P(X = 7)$.

c. Compute the expected value of X.

d. Compute the standard deviation for X. Is this value large in relation to the expected value? In what units is the standard deviation expressed?

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14. A company that makes traffic signal lights buys switches from a supplier. Out of each shipment of 1000 switches, the company will take a random sample of 10 switches. Let X equal the number of defective switches in the sample.

a. The company has a policy of rejecting a lot if they find any defective switches in the sample. What is the probability that the shipment will be accepted if, in fact, 2% of the switches are actually defective?

b. What is the probability that the shipment will be accepted if the percent of defective switches is actually 5%?

c. The company decides to change their policy and will accept the lot if they find no more than one defective switch. Repeat parts **a.** and **b.** for this new policy.

15. Parents have always wondered about the sex of a child before it is born. Suppose that the probability of having a male child was 0.5, and that the sex of one child is independent of the sex of other children.

a. Determine the probability of having exactly two girls out of four children.

b. What is the probability of having four boys out of four children?

16. A certain aspirin is advertised as being preferred by 4 out of 5 doctors. If the advertisement is assumed to be true, answer the following questions.

a. What is the probability that at least half of ten doctors chosen at random will prefer this brand of aspirin?

b. What is the probability that 9 out of 10 of the doctors will prefer this brand?

17. In manufacturing integrated circuits, the yield of the manufacturing process is the percentage of good chips produced by the process. The probability that an integrated circuit manufactured by the Ace Electronics Company will be defective is $p = 0.05$. If a random sample of 15 circuits is selected for testing, answer the following questions.

a. What is the probability that no more than one integrated circuit will be defective in the sample?

b. What is the expected number of defective integrated circuits in the sample?

18. The Alvin Secretarial Service procures temporary office personnel for major corporations. They have found that 90% of their invoices are paid within 10 working days. If a random sample of 12 invoices is checked, answer the following questions.

a. What is the probability that all of the invoices will be paid within 10 working days?

b. What is the probability that six or more of the invoices will be paid within 10 working days?

19. An experiment consists of rolling a pair of dice 10 times. On each roll the sum of the dots on the two dice is noted.

a. Find the probability that on any roll of the two dice the sum of the dots is either 7 or 11.

b. Find the probability that in the 10 rolls of the pair of dice, a 7 or 11 occurs 5 times.

c. Find the probability that in the 10 rolls of the pair of dice, a 7 or 11 does not occur at all.

d. Find the mean and variance of the number of times we see a 7 or 11 in the 10 rolls of the dice.

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20. *Would you say you eat to live or live to eat*? was asked to each person in a sample of 1001 adults in a Gallup Poll taken in April 1996. Seventy-four percent of the respondents answered eat to live, 23% answered live to eat, and 3% had no opinion. Assuming these percentages are accurate, find the probability, in 12 randomly chosen adults, that the number who would answer "eat to live" is:

a. exactly 7.

b. no more than 10.

c. at most 11.

d. at least 3.

6.5 The Poisson Distribution

The binomial random variable requires a fixed number of repetitions of the experiment, where the outcomes are either successes or failures. The **Poisson distribution** is similar to the binomial in that the random variable represents a count of the total number of successes. The major difference between the two distributions is that the Poisson does not have a fixed number of trials. Instead, the Poisson uses a fixed interval of time or space in which the number of successes are recorded. Thus, there is no theoretical upper limit on the number of successes, although large numbers of successes are not very likely. The word *success* in the Poisson context can sometimes take on rather unpleasant connotations. For example, the randomness exhibited by the number of airplane crashes, oil tanker spills, and car accidents in some fixed period of time seem to conform to the randomness described by a Poisson random variable.

In business environments many variables seem to follow a pattern of randomness similar to that described by the Poisson distribution. One of the Poisson's principal areas of use in business is the analysis of waiting lines. Other random phenomena, such as airplane arrivals at an airport, trucks arriving at a loading dock, users logging on to a computer system, or the number of defects in a given surface area, can be modeled with a Poisson distribution. These variables are often of interest in determining personnel requirements, inventories, and quality control.

Kicked by Horses

A real world example of the Poisson distribution involves the distribution of Prussian cavalry deaths from getting kicked by horses, in the period 1875–1894. The Prussian military kept meticulous records on horse-kick deaths in each of its army corps, and the data are neatly summarized in a 1963 book called Lady Luck, by the late Warren Weaver. There were a total of 196 kicking deaths—these being the successes. The trials were each army corps's observations on the number of kicking deaths sustained during the year. With 14 army corps and data for 20 years, there were 280 trials. The Poisson formula predicts, for example, that there will be 34.1 instances of having exactly two deaths in a year. In fact, there were 32 such cases. Pretty good, eh?

Procedure

Poisson Random Variable

In order to qualify as a **Poisson random variable** an experiment must meet two conditions.

1. Successes occur one at a time. (That is, two or more successes cannot occur at exactly the same point in time or exactly at the same point in space.)
2. The occurrence of a success in any interval is independent of the occurrence of a success in any other interval.

If these two conditions are met, it can be proven that the random variable for the number of successes follows the Poisson probability distribution function.

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