Table 6.2.10 – Return on Truck Purchase (cont.)

Return (Dollars)	Truck 1	Truck 2
15,000	0.20	0.30
20,000	0.03	0.20
25,000	0.02	0.05

Calculate the expected return and standard deviation for each truck and recommend which truck to purchase.

Table 6.2.11 – Calculations

Return (Dollars)	Truck 1	Truck 2	Truck 1 $xp(x)$	Truck 2 $xp(x)$	Truck 1 $(x-\mu)^2 \cdot p(x)$	Truck 2 $(x-\mu)^2 \cdot p(x)$
−5000	0.02	0.15	−100	−750	4,500,000	36,037,500
0	0.03	0.10	0	0	3,000,000	11,025,000
5000	0.20	0.10	1000	500	5,000,000	3,025,000
10,000	0.50	0.10	5000	1000	0	25,000
15,000	0.20	0.30	3000	4500	5,000,000	6,075,000
20,000	0.03	0.20	600	4000	3,000,000	18,050,000
25,000	0.02	0.05	500	1250	4,500,000	10,512,500
		$E(X) =$	$10,000	$10,500		
				$V(X) = \sigma^2 =$	25,000,000	84,750,000
				$\sqrt{V(X)} = \sigma =$	$5000	$9206

The expected value and standard deviation of purchasing Truck 1 are $10,000 and $5000, respectively. Similarly, the expected value and standard deviation of purchasing Truck 2 are $10,500 and $9206. Given that the risk (standard deviation) of Truck 2 is nearly twice that of Truck 1, it appears that selecting Truck 1 is the best decision, even though the expected return is slightly less.

6.2 Exercises

Basic Concepts

1. Discrete probability distributions always have three characteristics. What are they?
2. What is the value of describing a random variable with a probability distribution?
3. What are three different ways to express possible values of a random variable along with their associated probabilities?
4. How is a probability distribution created?
5. Identify four discrete probability distributions.
6. What is a probability distribution function?
7. Why is the notion of expected value important in the analysis of random phenomena?
8. True or false: the expected value of a random variable is usually one of the possible outcomes of the random variable.
9. Suppose the expected value of a random variable was known to be 6.3. Interpret the meaning of the expected value.

10. Give an example of a situation in which expected value would be useful to compare alternatives.

11. How is the variance (or standard deviation) of a random variable related to risk?

Exercises

12. Determine whether or not the following distribution is a probability distribution. If the distribution is not a probability distribution, give the characteristic which is not satisfied by the distribution.

x	$P(X = x)$
1	$\frac{1}{3}$
2	$\frac{2}{3}$
3	$\frac{1}{3}$

13. Determine whether or not the following distribution is a probability distribution. If the distribution is not a probability distribution, give the characteristic which is not satisfied by the distribution.

x	$P(X = x)$
–2	0.25
2	0.50
3	0.25

14. Determine whether or not the following distribution is a probability distribution. If the distribution is not a probability distribution, give the characteristic which is not satisfied by the distribution.

x	$P(X = x)$
2	0.30
3	–0.50
4	0.50
5	0.70

15. Determine whether or not the following distribution is a probability distribution. If the distribution is not a probability distribution, give the characteristic which is not satisfied by the distribution.

x	$P(X = x)$
5	0.46
10	0.25
15	0.25

16. Determine whether or not the following distribution is a probability distribution. If the distribution is not a probability distribution, give the characteristic which is not satisfied by the distribution.

x	$P(X = x)$
–10	0.18
–5	0.39
3	0.08
8	0.35

17. Determine whether or not the following distribution is a probability distribution. If the distribution is not a probability distribution, give the characteristic which is not satisfied by the distribution.

x	$P(X = x)$
100	–0.10
200	0.50
300	0.50

18. Determine whether or not the following distribution is a probability distribution. If the distribution is not a probability distribution, give the characteristic which is not satisfied by the distribution.

$$P(X = x) = \frac{x}{16}, \text{ for } x = 1,\ 2,\ 3,\ 4,\ 5$$

19. Determine whether or not the following distribution is a probability distribution. If the distribution is not a probability distribution, give the characteristic which is not satisfied by the distribution.

$$P(X = x) = \frac{x^2}{30}, \text{ for } x = 1,\ 2,\ 3,\ 4$$

20. Find the expected value, the variance, and the standard deviation for a random variable with the following probability distribution.

x	-5	-2	0	2	5
$p(x)$	0.06	0.15	0.58	0.18	0.03

21. Find the expected value, the variance, and the standard deviation for a random variable with the following probability distribution.

x	400	420	440	460	480	500
$p(x)$	0	0.1	0.1	0.2	0.2	0.4

22. A regional hospital is considering the purchase of a helicopter to transport critical patients. The relative frequency of X, the number of times the helicopter is used to transport critical patients each month, is derived for a similarly sized hospital and is given in the following probability distribution.

Number of Helicopter Transports							
x	0	1	2	3	4	5	6
$p(x)$	0.15	0.20	0.34	0.19	0.06	0.05	0.01

a. Find the average number of times the helicopter is used to transport critical patients each month.

b. Find the variance of the number of times the helicopter is used to transport critical patients.

c. Find the standard deviation of the number of times the helicopter is used to transport critical patients.

d. Find the probability that the helicopter will not be used at all during a month to transport critical patients.

e. Find the probability that the helicopter will be used at least once to transport critical patients.

f. Find the probability that the helicopter will be used at most twice to transport critical patients.

g. Find the probability that the helicopter will be used more than three times to transport critical patients.

23. Based on past experience, an architect has determined a probability distribution for X, the number of times a drawing must be examined by a client before it is accepted.

Number of Times Examined					
x	1	2	3	4	5
$p(x)$	0.1	0.2	0.3	0.2	0.2

a. Find the average number of times a drawing must be examined by a client before it is accepted.

b. Find the variance of the number of times a drawing must be examined by a client before it is accepted.

c. Find the standard deviation of the number of times a drawing must be examined by a client before it is accepted.

d. What is the probability that a drawing must be examined five times before being accepted by the client?

e. Find the probability that the drawing must be examined at least twice before being accepted by the client.

f. Find the probability that a drawing must be examined at most three times before being accepted by the client.

g. Find the probability that a drawing must be examined less than twice before being accepted by the client.

24. The manager of a retail clothing store has determined the following probability distribution for X, the number of customers who will enter the store on Saturday.

Customers on Saturday						
x	10	20	30	40	50	60
$p(x)$	0.10	0.20	0.30	0.20	0.10	0.10

a. Find the expected number of customers who will enter the store on Saturday.

b. Find the standard deviation of the number of customers who will enter the store on Saturday.

c. Find the variance of the number of customers who will enter the store on Saturday.

d. Find the probability that more than 30 customers will enter the store on Saturday.

e. Find the probability that at most 20 customers will enter the store on Saturday.

f. Find the probability that at least 40 customers will enter the store on Saturday.

g. What is the probability that exactly 10 customers will enter the store on Saturday?

25. An entrepreneur is considering investing in a new venture. If the venture is successful, he will make \$50,000. However, if the venture is not successful, he will lose his investment of \$10,000. Based on past experience, he believes that there is a 40% chance that the venture will be successful.

a. Use the information in the problem to determine the probability distribution of the amount of money to be made (or lost) on the venture.

b. Determine the expected amount of money to be made on the venture.

c. Determine the standard deviation of the amount of money to be made on the venture.

26. An investor is considering two alternative investment options with the following payoff distributions.

	Option 1			Option 2		
Payoff	–$100,000	$30,000	$100,000	–$20,000	$0	$20,000
***P*(Payoff)**	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	0.25	0.50	0.25

a. Calculate the expected payoff for each of the investment options.

b. Calculate the standard deviation of the payoff for each of the investment options.

c. Which investment option would you choose? Explain.

27. A cereal manufacturer has two new brands of cereal which it would like to produce. Because resources are limited, the cereal manufacturer can only afford to produce one of the new brands. A marketing study produced the following probability distributions for the amount of sales for each of the new brands of cereal.

Cereal A		Cereal B	
Sales	***P*(Sales)**	**Sales**	***P*(Sales)**
$150,000	0.2	$10,000	0.40
$200,000	0.3	$300,000	0.40
$300,000	0.3	$600,000	0.10
$400,000	0.2	$1,000,000	0.10

a. What are the expected sales of each of the new brands of cereal?

b. What is the standard deviation of the sales for each of the brands of cereal?

c. If both of the brands of cereal cost the same amount to produce, which brand of cereal do you think the cereal manufacturer should produce? Explain.

6.3 The Discrete Uniform Distribution

The **discrete uniform distribution** is one of the simplest probability distributions. Each value of the random variable is assigned an identical probability. There are many situations in which the discrete uniform distribution arises. Some common examples are rolling a fair die or flipping a fair coin.

Formula

Discrete Uniform Probability Distribution Function

Mathematically, the **discrete uniform probability distribution function** is given by

$$P(X = x) = \frac{1}{n}$$

where n = the number of values that the random variable may assume.

Example 6.3.1

Determining the Probability Distribution of Throwing a Die

What is the probability distribution for the outcome of the throw of a single six-sided die?

SOLUTION

If the die is fair, then each of the outcomes is equally likely, and thus we have a discrete uniform distribution in which all probabilities equal $\frac{1}{6}$. The probability distribution is given in Table 6.3.1.