From the items calculated in parts **a.** and **b.**, we know

$$\begin{aligned}P(P \mid B) &= \frac{P(P \cap B)}{P(B)} = \frac{P(B \mid P)P(P)}{P(B)} \\ &= \frac{P(B \mid P)P(P)}{P(B \mid M)P(M) + P(B \mid P)P(P) + P(B \mid C)P(C)} \\ &= \frac{(0.6)(0.3)}{0.57} \\ &\approx 0.3158.\end{aligned}$$

Thus, we know that if the passenger is traveling on business, there is about a 32% chance that he or she will be traveling by private plane.

Even though it was fairly subtle (given that we performed the calculations in parts **a.** and **b.**), please note the use of Bayes' theorem in the previous calculation.

5.5 Exercises

Basic Concepts

1. Briefly explain the relationship between conditional probability and Bayes' theorem.

2. Other than conditional probability, which other rule which you previously studied is used in the derivation of Bayes' theorem?

3. What is Bayes' theorem?

4. How is Bayes' theorem used to "revise" a probability based on additional information?

Exercises

5. The issue of Corporate Tax Reform has been cause for much debate in the United States, especially in the House Ways and Means Committee as well as the Senate Finance Committee. Among those in the legislature, 45% are Republicans and 55% are Democrats. It is reported that 30% of the Republicans and 70% of the Democrats favor some type of Corporate Tax Reform to prevent American companies from operating in foreign countries. Suppose a member of Congress is randomly selected and they are found to favor some type of corporate tax reform. What is the probability that this person is a Democrat?

6. Adults (18 years and older) and kids (under 13 years of age) are observed to react differently to sad, emotional movies. It has been observed that 70% of the kids say they cry at some point during those types of movies, whereas only 40% of the adults admit to crying during those types of movies. A group of 40 people, of whom 25 are kids, was shown a sad, emotional movie and the subjects were asked if they cried. A response picked at random from the 40 indicated that they cried. What is the probability that it was an adult?

7. As items come to the end of a production line, an inspector chooses which items are to go through a complete inspection. Eight percent of all items produced are defective. Sixty percent of all defective items go through a complete inspection, and 20% of all good items go through a complete inspection. Given that an item is completely inspected, what is the probability that it is defective?

8. Two teaching methods for a business statistics class, online and face-to-face, are available during the course of an academic year. The failure rate (students that receive below a C– and thus, will have to repeat the course) is 4% for the online class and 8% for the face-to-face class. However, the online class is more expensive and hence is offered only 25% of the time. (The face-to-face class is offered the other 75% of the time.) A student takes the statistics class via one of the methods of delivery but failed the course. What is the probability that the student took the online class?

9. A personnel director has two lists of applicants for jobs. List 1 contains names of 15 women and 5 men whereas List 2 contains the names of 5 women and 12 men. A name is randomly selected from List 1 and added to List 2. A name is then randomly selected from the augmented List 2. Given that the name selected is that of a man, what is the probability that a woman's name was originally selected from List 1?

5.6 Counting Techniques

To compute certain probabilities, such as the probability of having winning numbers in the state lottery, requires the ability to count the number of possible outcomes for a given experiment or a sequence of experiments.

However, often it is impractical to list out all the possibilities. Therefore, we will develop some techniques to facilitate our counting.

The Fundamental Counting Principle

Theorem

Fundamental Counting Principle

E_1 is an event with n_1 possible outcomes and E_2 is an event with n_2 possible outcomes. The number of ways the events can occur in sequence is $n_1 \cdot n_2$. This principle can be applied for any number of events occurring in sequence.

Example 5.6.1

Using the Fundamental Counting Principle to Count Employees

A local bank has three branches. Each branch has four departments and each department has two employees. How many employees does the bank have?

SOLUTION

$$\underset{\text{(number of branches)}}{3} \cdot \underset{\text{(departments)}}{4} \cdot \underset{\text{(employees per department)}}{2} = \underset{\text{(total number of employees)}}{24}$$

Thus, the bank has 24 total employees.

Example 5.6.2

Using the Fundamental Counting Principle to Count License Plates

Nonpersonalized license plates in the state of Utah consist of three numbers followed by three letters (excluding I, O, and Q). How many license plates are possible?

SOLUTION

There are ten digits (0 – 9) possible for each of the first three characters. Likewise, there are 23 letters possible for the last three characters. Therefore, we have the following.

$$\underset{\text{(digit)}}{10} \cdot \underset{\text{(digit)}}{10} \cdot \underset{\text{(digit)}}{10} \cdot \underset{\text{(letter)}}{23} \cdot \underset{\text{(letter)}}{23} \cdot \underset{\text{(letter)}}{23} = \underset{\text{possible license plates}}{12,167,000}$$