

## 5.4 Exercises

### Basic Concepts

1. Explain the difference between dependent and independent events.
2. Are mutually exclusive events dependent or independent? Explain your answer.
3. If events  $A$  and  $B$  are independent, what is  $P(A|B)$  equal to?
4. What is the product rule?
5. In the case *People v. Collins* an appeals court overturned the conviction. What flaws did the appeals court detect in the case against the accused assailants?

### Exercises

6. The following table was given in Section 5.2, Exercise 12.

Health Care Consumers		
Have Health Insurance Coverage	Housing Situation	
	Rent	Own
Yes	196	298
No	92	173

Are the events {customer rents their home} and {customer owns their home} independent? Explain.

7. The following table was given in Section 5.2, Exercise 13.

Life Insurance Coverage					
		Amount of Life Insurance on Husband (\$)			
		0 – 50,000	50,000 – 100,000	100,000 – 150,000	More than 150,000
Amount of Life Insurance on Wife (\$)	0 – 50,000	400	200	50	50
	50,000 – 100,000	50	50	30	30
	100,000 – 150,000	20	10	25	25
	More than 150,000	20	10	15	15

Are the events {the husband has more than \$150,000 in insurance} and {the wife has more than \$50,000 in insurance} independent? Explain.

8. Suppose you were flipping a coin. What is the probability that you would observe a head:
  - a. on two consecutive flips?
  - b. on three consecutive flips?
  - c. on four consecutive flips?
  - d. on 100 consecutive flips?
9. Suppose an atomic reactor has two independent cooling systems. The probability that Cooling System A will fail is 0.01 and the probability that Cooling System B will fail is 0.01. What is the probability that both systems will fail simultaneously?
10. Mandy is 30, and the probability that she will survive until age 65 is 0.90. Ashley is 45, and the probability that she will survive until age 65 is 0.95.
  - a. Find the probability that both Mandy and Ashley will survive until age 65.
  - b. Find the probability that only Mandy will survive until age 65.
  - c. Find the probability that neither Mandy nor Ashley will survive until age 65.
  - d. What assumption about the lives of Mandy and Ashley did you make in answering the above questions?

11. An insurance company is considering insuring two large oil tankers against spills. The limit of the liability on the coverage is \$10,000,000. The company believes that the probability of an oil spill requiring the maximum liability coverage during the policy period is 0.001 per tanker.
  - a. What is the probability that neither tanker would have a spill requiring the maximum liability coverage during the policy period?
  - b. What is the probability that only one tanker would have a spill requiring the maximum liability coverage during the policy period?
  - c. What is the probability that both tankers would have spills requiring the maximum liability coverage during the policy period?
12. Coin flipping can be used to model other real-life phenomena and aid in certain probability calculations. An example of this would be to compute the probability that the World Series ends in some specified number of games. The World Series is a best of seven game series played at the end of the regular baseball season between the champion of the American League and the champion of the National League. The first team to win four games is declared the champion of baseball for that year. If we assume the probability of either team winning a game is approximately 0.5 and the games are independent events, the probability that the series ends in either 4, 5, 6, or 7 games can be computed.
  - a. What is the probability that the series ends in exactly 4 games? Write the sample space consisting of 16 equally likely simple events similar to the sample space resulting from tossing a coin four times.
  - b. What is the probability that the series ends in exactly 5 games?
  - c. Assume the probability that the series ends in exactly 6 games is  $\frac{5}{16}$ . Use this information together with your answers to the first two parts of this problem to compute the probability that the series ends in exactly 7 games.
13. Drug usage in the workplace costs employers incredible amounts of money each year. Drug testing potential employees has become so prevalent that drug users are finding it extremely hard to find jobs. Drug tests, however, are not completely reliable. The most common test used to detect drugs is approximately 98% accurate. To decrease the likelihood of making an error, all potential employees are screened through two tests, which are independent, and each has about 98% accuracy.
  - a. If a person were drug free, what is the probability he or she would fail both tests?
  - b. If a person were a drug user, what is the probability he or she would pass both tests?

## 5.5 Bayes' Theorem

We have completed the discussions about conditional probability and independent events in Section 5.3 and Section 5.4. **Bayes' theorem** (also referred to as **Bayes' rule** or **Bayes' law**) is somewhat of an extension of conditional probability in which we calculate probabilities based on new information. Please note and understand that the additional information is obtained for a subsequent event, and the new information is used to revise the initial probability. Recall the following formulas for conditional probability.