Then the desired probability can be formulated as

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}.$$

The $P(A \cap B)$ is called a joint probability since it is the probability of the occurrence of more than one event. To compute $P(A \cap B)$ use the empirical approach.

$$P(A \cap B) = \frac{193}{1403} \approx 0.1376$$

Similarly, $P(B)$ can be computed as

$$P(B) = \frac{444}{1403} \approx 0.3165.$$

Consequently, $P(A \mid B)$ is

$$P(A \mid B) \approx \frac{0.1376}{0.3165} \approx 0.4348.$$

Note that this answer could have also been obtained by simply dividing 193 by 444.

5.3 Exercises

Basic Concepts

1. Define conditional probability.
2. How do you calculate $P(A \mid B)$?

Exercises

3. The following table was given in Section 5.2, Exercise 12.

Health Care Consumers

Have Health Insurance Coverage	Housing Situation: Rent	Housing Situation: Own
Yes	196	298
No	92	173

   a. Given that the customer rents their home, what is the probability that the customer does not have health insurance?

   b. Given that the customer does not have health insurance, what is the probability that the customer rents their home?

   c. Given that the customer owns their home, what is the probability that the customer has health insurance?

   d. Given that the customer has health insurance, what is the probability that the customer owns their home?

4. The following table was given in Section 5.2, Exercise 13.

Life Insurance Coverage

		Amount of Life Insurance on Husband ($)			
		0 – 50,000	50,000 – 100,000	100,000 – 150,000	More than 150,000
Amount of Life Insurance on Wife ($)	0 – 50,000	400	200	50	50
	50,000 – 100,000	50	50	30	30
	100,000 – 150,000	20	10	25	25
	More than 150,000	20	10	15	15

Let's Make a Deal

A long time ago, back in the 70s, there was a television show called "Let's Make a Deal" starring Monty Hall as the host. This show produced an interesting problem in probability which someone submitted to Marilyn vos Savant which she answered in her column in Parade magazine. Incidentally, Ms. Savant is in the Guinness Book of World Records as having the highest recorded IQ (228). Here's the problem that was posed to Ms. Savant.

"Suppose you're on a game show, and you're given a choice of three doors: Behind one door is a car: behind the others, goats. You pick a door, say number 1, and the host, who knows what's behind the other doors, opens another door, say number 3, which has a goat. He then says to you, 'Do you want to pick door number 2?' Is it to your advantage to take the switch?"

Marilyn vos Savant answered the question in her column saying that it was to your advantage to switch. This set off a firestorm of mail telling Ms. Savant that she was incorrect. Much of this mail came from people with Ph.D.s behind their names. The New York Times printed a front page article in 1991 discussing the problem.

What do you think? To find the answer to this problem type "The Monty Hall problem" in your search engine and go to some of the web sites and try some of the simulations.

a. Given the wife has between \$100,000 and \$150,000 of insurance, what is the probability that the husband has more than \$150,000 of insurance?

b. Given the wife has between \$0 and \$50,000 of insurance, what is the probability that the husband has between \$0 and \$150,000 of insurance?

c. Given that the husband has between \$0 and \$50,000 of insurance, what is the probability that the wife will have more than \$150,000 of insurance?

d. Given that the husband has more than \$150,000 of insurance, what is the probability that the wife will have more than \$150,000 of insurance?

5. A computer software company receives hundreds of support calls each day. There are several common installation problems, call them A, B, C, and D. Several of these problems result in the same symptom, *lock up* after initiation. Suppose that the probability of a caller reporting the symptom *lock up* is 0.7 and the probability of a caller having problem A and a *lock up* is 0.6.

a. Given that the caller reports a lock up, what is the probability that the cause is problem A?

b. What is the probability that the cause of the malfunction is not problem A given that the caller is experiencing a lock up?

6. A television advertising representative has determined the following probabilities based on past experience. The probability that an individual will watch an ad during the Super Bowl is 0.10. Given that the individual watches the ad, the probability that the individual will buy the product is 0.005. It is also known that the probability that an individual would buy the product is 0.02. Given that an individual buys the product, find the probability that the individual watched the television ad during the Super Bowl.

7. Medical researchers have determined that there is a 2% chance that an individual will have a gene which gives him a predisposition for heart disease. Given that an individual has the gene, the probability that heart disease will develop is 25%. It is also known that the probability that an individual has heart disease is 12%.

a. Find the probability that an individual will have the gene and develop heart disease.

b. Given that a person has heart disease, what is the probability that they have the gene?

5.4 Independence

An extremely important concept in statistical analysis is **independence**. It describes a special kind of relationship between two events. Two events are said to be independent if knowledge of one event does not provide information of the other event's occurrence. In other words, the occurrence of one event does not affect the occurrence of another event if the events are independent.

Example 5.4.1

Determining the Independence of Events

Experiment: roll a fair die two times. Consider the two events

A = {rolling a six on the first roll of a fair die} and

B = {rolling a four on the second roll of a fair die}.

Are these two events independent?

SOLUTION

Since knowledge of the outcome of the first roll does not help one make an inference of the outcome of the second roll, events A and B are independent.