We want to know the probability of event A or event B or both. Thus, the desired probability is the union of the two events given by

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.2 + 0.3 - 0.08 = 0.42.$$

Therefore, the probability of finding someone who is earning over \$50,000 or subscribes to more than one sports magazine, or both, is 0.42.

5.2 Exercises

Basic Concepts

1. What laws must probability obey, regardless of the methodology used to derive the probabilities?

2. Suppose you are taking a test next week. Interpret each of the following statements.

- **a.** P(receiving an A on the test) = 0
- **b.** P(receiving an A on the test) = 1
- **c.** P(receiving an A on the test) = 0.3

3. What is a compound event?

4. Define the following set operations: union, intersection, and complement.

5. If you know the probability of two events, what else must you know in order to calculate the probability of *one event or the other*?

6. If events A and B are mutually exclusive, what is $P(A \cap B)$?

Exercises

7. Determine if the following values could be probabilities. If the value cannot be a probability, explain why.

- **a.** 0
- **b.** $\frac{36}{25}$
- **c.** $\frac{7}{8}$
- **d.** –0.4
- **e.** 0.23

8. Determine if the following values could be probabilities. If the value cannot be a probability, explain why.

- **a.** 1
- **b.** $\frac{15}{16}$
- **c.** $\frac{4}{3}$
- **d.** 0.99
- **e.** –0.05

9. Interpret the following probabilities with respect to the occurrence of some event.

- **a.** $P(\text{event}) = 0$
- **b.** $P(\text{event}) = 1.0$
- **c.** $P(\text{event}) = 0.45$
- **d.** $P(\text{event}) = 65\%$
- **e.** $P(\text{event}) = -1.0$

10. Find the following probabilities.

- **a.** The probability of an event that must happen.
- **b.** The probability of an event that cannot happen.
- **c.** The probability of having a boy or a girl in a single birth.
- **d.** The probability of rolling a two and a five in a single toss of a die.

11. The annual premium amounts charged by life insurance companies to their clients are set very carefully. If the amount is too high, the client will take his or her business to another company. If it is too low, the insurance company may not make enough profit to stay in business. In order to properly determine a premium, the company often relies on life tables. These tables allow one to compute the probabilities of death at various ages. They are constructed only after collecting and reviewing extensive data on age at death from a large group of people. A life table is normally constructed assuming that 100,000 people are alive at age 0. This number is simply a reference value used to make comparisons throughout the table. Other numbers could be used. The table then gives the number of people of the original 100,000 that are alive at the beginning of various years of life. In order for the insurance company to optimally set premiums, a separate table should be constructed for the different genders and races. The following abbreviated life table is valid only for females.

Life Table

Year	0	1	5	10	15	20	25	30	35
Number Alive	100,000	99,090	98,912	98,815	98,716	98,477	98,204	97,897	97,500
Year	40	45	50	55	60	65	70	75	80
Number Alive	96,958	96,097	94,766	92,623	89,449	84,565	77,772	68,200	55,535

a. What is the probability that a newborn female lives until the age of 40?

b. What is the probability that a newborn female dies before she reaches the age of 50?

12. A health care provider classifies its customers by their housing situation and whether they have health insurance coverage. The market research department has gathered data from a random sample of 759 customers.

Health Care Consumers

	Housing Situation	
Have Health Insurance Coverage	**Rent**	**Own**
Yes	196	298
No	92	173

a. What is the probability that a customer rents their home?

b. What is the probability that a customer owns their home?

c. What is the probability that a customer has health insurance coverage and rents their home?

d. What is the probability that a customer owns their home and does not have health insurance coverage?

e. What is the probability that a customer has health insurance coverage and rents their home or does not have health insurance coverage and owns their home?

f. What is the probability that a customer does not have health insurance coverage?

g. What approach to probability did you use to calculate your answers?

h. Are the events {rents their home} and {owns their home} mutually exclusive? Explain.

13. A large life insurance company is interested in studying the insurance policies held by married couples. In particular, the insurance company is interested in the amount of insurance held by the husbands and the wives. The insurance company collects data for all of its 1000 policies where both the husband and the wife are insured. The results are summarized in the following table.

Life Insurance Coverage

		Amount of Life Insurance on Husband ($)			
		0 – 50,000	50,000 – 100,000	100,000 – 150,000	More than 150,000
Amount of Life Insurance on Wife ($)	0 – 50,000	400	200	50	50
	50,000 – 100,000	50	50	30	30
	100,000 – 150,000	20	10	25	25
	More than 150,000	20	10	15	15

a. For a randomly selected policy, what is the probability that the husband will have between $50,000 and $100,000 of insurance?

b. For a randomly selected policy, what is the probability that the wife will have between $100,000 and $150,000 of insurance?

c. For a randomly selected policy, what is the probability that the wife will have more than $150,000 of insurance or the husband will have more than $150,000 of insurance?

d. For a randomly selected policy, what is the probability that the wife will have between $0 and $50,000 of insurance and the husband will have between $0 and $50,000 of insurance?

e. For a randomly selected policy, what is the probability that the wife will not have between $0 and $50,000 of insurance?

f. For a randomly selected policy, what is the probability that the husband will have more than $50,000 of insurance?

g. What approach to probability did you use to calculate your answers?

h. Are the events {the wife has more than $150,000 in insurance} and {the husband has between $50,000 and $100,000 of insurance} mutually exclusive? Explain.

5.3 Conditional Probability

Researchers often want to examine a limited portion of the sample space. For example, consider the question of whether cigarette smoking harms those that are indirectly exposed to the smoke. Suppose that 3 percent of women who do not smoke die of cancer. However, if a nonsmoking woman is married to a smoking husband (not to be confused with a husband who is on fire), the probability of dying of cancer is 0.08. This probability is a **conditional probability**, because the sample space is being limited by some condition—in this case, limited to only wives of smoking husbands. In this instance, the dramatic effect of a smoking husband on cancer rates is readily evident.

> **Definition**
>
> **Conditional Probability**
>
> The probability that one event will occur given that some other event has occurred is a **conditional probability**.

$$P(\text{a nonsmoking woman dies of cancer})$$

$$\neq$$

$$P(\text{a nonsmoking woman dies of cancer given that her husband smokes})$$

Similarly, the results from a market survey indicate that 39 percent of the customers surveyed believe a product is of high quality. However, if the analysis is limited to only women, 54