Definition

Statistical Inference

The process of making judgments about population parameters is called **statistical inference**.

Lloyd's of London

This very modern looking building is the home of the world's second largest commercial insurer and the sixth largest reinsurance group, Lloyd's of London. At Lloyd's, like all other insurers, risk is measured in probabilities, which are usually subjective. Lloyd's differs from other insurers in the kinds of policies they write. Lloyd's has written policies on nuclear reactors, space shuttle cargo, oil tankers, art treasures, kidnap and ransom, as well as the legs of ballerinas and football players.

Insurance has a very important place in commerce, and without it, many business activities would not be possible. If a shipping company could not insure its ships, raising the money to buy them would be virtually impossible. Insurance is big business. Lloyd's annual marine insurance premiums amount to more than $30 billion a year, and that represents little more than one-third of their aggregate income. In addition to sizable revenues, Lloyd's employs about 70,000 people. Lloyd's is a market, rather than an entity. It houses underwriters who evaluate insurance risk for the syndicates they represent. A syndicate is a group of individuals, called Names, who individually assume a small amount of risk in return for a commensurate portion of the premium. To become a Name you must have a net worth in excess of $550,000 (excluding the value of your home) and apply to Lloyd's committee for approval. For large policies, like an ocean cargo vessel, even a syndicate does not usually underwrite the entire policy; more often groups of syndicates each take a small percentage–thus further diluting each individual's risk.

Criticism of the Subjective View

If science is defined as finding out what is probably true, there should be a probability criterion on which all reasonable persons could agree. But if probability is subjective, how can it be used as a universally accepted criterion? Two reasonable persons might examine the same data and reach different conclusions about their degree of belief about some proposition.

Probability, Statistics, and Business

Most of the time, when working with samples, statisticians try to deduce from the samples the population parameters (means, proportions, variances, etc.) of certain variables. This process of making judgments about population parameters is called **statistical inference**. Because samples are random, there is no guarantee that the sample will be representative of the population. If the sample is not representative, then using the sample mean as an estimate (inference) of the population mean would not be very wise. Probability is used to assess the quality of our inference. All statistical conclusions must be endowed with a degree of uncertainty. Because probability is used to assess the reliability of sample inferences, it is the foundation of all inferential statistics.

The probability concept also has many direct applications in business. When a manager wonders whether dropping a bid price by 5% will increase the probability of winning the bid, he or she is thinking about chance. Probability is also used as a criterion in designing and evaluating product reliability, evaluating insurance, inventory management, project management, and in the study of queuing theory (a probabilistic analysis of waiting lines).

Probability theory emerged from the need to better understand a game of chance. Business decisions, like games, have uncertain outcomes. In an effort to make better decisions, businesses spend considerable amounts of money trying to quantify uncertainty. This means trying to turn uncertainty into a probability. Insurance companies have historically done a good job of quantifying uncertainty. In fact, a special kind of statistician called an actuary has emerged to assist in the development of insurance models which quantify uncertainty and aid in business decisions.

For example, the next time you watch a 30-second commercial during the Super Bowl, consider the fact that a company has just spent roughly $3 million for the airtime plus a substantial amount of money developing the advertisement. Without knowing the effect of the advertisement in advance, extensive amounts of money are put at risk with an uncertain outcome. The manager making the decision uses subjective probability to assess the risk and reward.

5.1 Exercises

Basic Concepts

1. Describe randomness.
2. What is probability?
3. What are the conditions of a random experiment?
4. Consider the random experiment of flipping a fair coin twice. What is the sample space for this experiment?
5. What is an event?
6. Consider the random experiment of rolling a fair die once. Give an example of an event for this experiment and list the outcomes associated with that particular event.
7. What are the two main branches of probability?

8. What are the two approaches to objective probability?

9. What are some of the problems associated with the relative frequency approach?

10. True or false: According to the mathematical law of probability, the observed relative frequency of heads when flipping a coin will eventually reach 0.5 since the probability of heads is 0.5.

11. What is statistical regularity?

12. Describe the classical approach to probability.

13. Using the classical approach, describe how you would determine the probability of event A.

14. What is the subjective approach to probability? Discuss the problems of applying the subjective interpretation.

15. What is statistical inference?

16. Discuss the relationship between probability and statistics.

17. Give three applications of probability in business.

18. Describe the importance of probability in the insurance industry.

19. What type of probability does the manager of a company use when purchasing a commercial spot during the Super Bowl? Explain why.

Exercises

20. Consider the following random experiment. A potato chip manufacturer is interested in determining if the brand of potato chip which it manufactures is preferred over three of its major competitors. Several customers are randomly selected and asked which brand of potato chip they prefer: Brand A, Brand B, Brand C, or Brand D.

a. Determine the sample space for the experiment described.

b. If the manufacturer makes Brand A, list the outcomes in the event M = {customer does not prefer the manufacturer's brand}.

21. Consider the following random experiment. A doctor is interested in determining whether or not his patients think that he listens attentively to what they are saying. He randomly selects several patients and administers an anonymous survey that asks which of the following categories best describes his attentiveness: Very Attentive, Somewhat Attentive, Not Attentive.

a. Determine the sample space for the above experiment.

b. Determine all possible outcomes for the event A = {the doctor is not described as very attentive}.

22. A gambler has made a weighted die. In order to decide which of the six sides is most likely to turn up, he tosses the die 33 times and notes the number of dots on the upper-most surface. The results of the experiment are shown in the following table.

Rolls of a Weighted Die										
1	2	1	3	1	4	1	5	6	3	1
3	1	5	1	2	1	3	1	2	1	2
2	1	3	5	1	2	1	2	1	4	6

a. Using the relative frequency approach, what is the probability of observing each side?

b. Which side do you think the gambler will bet on when the die is tossed?

23. Assume there are two red, two yellow, and two blue buttons in a hat. A button is drawn out of the hat, the color is noted, and the button is returned. This is repeated fifty times. The results are listed in the following table.

Button Drawing				
Yellow	Yellow	Red	Yellow	Red
Red	Red	Blue	Red	Blue
Blue	Red	Red	Yellow	Red
Red	Blue	Yellow	Red	Yellow
Yellow	Blue	Red	Blue	Red
Red	Red	Red	Red	Yellow
Blue	Yellow	Yellow	Blue	Red
Yellow	Red	Red	Red	Yellow
Red	Yellow	Yellow	Yellow	Red
Red	Red	Blue	Red	Blue

Using the relative frequency approach, what is the probability of drawing each color?

24. Twenty-five insurance agents are randomly selected and asked if they own a handgun. Twenty-two of those surveyed said that they do own a handgun. If an insurance agent is randomly selected, estimate the probability that the agent will own a handgun.

25. Thirty elementary school teachers are randomly selected and asked if they favor standardized testing of elementary school children. Twenty of those surveyed said that they did favor standardized testing of elementary school children. If an elementary school teacher is randomly selected, estimate the probability that the teacher will favor standardized testing for elementary school children.

26. Fifty chief executive officers (CEOs) of publicly traded companies are randomly selected and their salaries are determined. Forty-five of the CEOs selected have salaries in excess of $500,000. If a CEO from one of the selected publicly traded companies is randomly selected, find the probability that the CEO will have a salary in excess of $500,000.

27. Forty emergency calls to which a local police department responded were randomly selected. Of the forty emergency calls fifteen were categorized as domestic arguments. Estimate the probability that the next emergency call to which the local police department responds will be a domestic argument.

28. For the following situations, decide which probability interpretation is most reasonable to use: relative frequency, subjective, or classical.

- **a.** Whether or not you will have a wreck on your next trip to the mall.
- **b.** Whether or not a car coming off the Ford assembly line will have a defect.
- **c.** The probability that you will graduate from college in four calendar years.
- **d.** Whether a person will be in an automobile accident during the next year.
- **e.** The probability that you will be dealt a full house from a well-shuffled deck of cards.

29. For the following situations, decide which probability interpretation is most reasonable to use: relative frequency, subjective, or classical.

- **a.** Suppose you have purchased a lottery ticket. Describe your chances of winning the lottery.
- **b.** The probability you will enjoy a vacation trip to Mexico.
- **c.** The probability your company's sales will exceed seven million dollars this year.

d. One hundred people receive keys to a new car in a radio contest. Only one key actually fits the car. The probability that key number 25 will open the car door.

e. The probability that you will get a ticket if you drive 70 mph on the interstate between work and home this coming Tuesday.

f. The probability that the S&P 500 will increase or decrease by at least 25 points in one day.

30. A couple plans to have two children.

a. List all possible outcomes for the sexes of the two children.

b. Find the probability that the couple will have 2 boys.

c. Find the probability that the couple will have at least 1 girl.

31. Consider a student who is taking a multiple choice examination where there are five possible answers for each question. Since the student has not studied or attended any of the classes, the student decides to randomly guess at each question.

a. Find the probability that the student will answer the first question correctly.

b. Find the probability that the student will answer the first question incorrectly.

32. A game show contestant has to choose one of three doors to win a prize. Behind one door the prize is a trip to Hawaii; behind another door, the prize is a color TV; behind the final door, the prize is a bag of potatoes. If a contestant randomly selects a door,

a. Find the probability that the contestant will win a trip to Hawaii.

b. Find the probability that the contestant will not win a trip to Hawaii.

5.2 Laws of Probability

Interpreting probability using the classical approach is a good way of thinking about the basic probability principles. In this section we will discuss certain laws that probabilities must obey, regardless of how probability is defined.

Probability Law 1

A probability of zero means the event cannot happen.

For example, the probability of observing three heads in two tosses of a coin is zero.

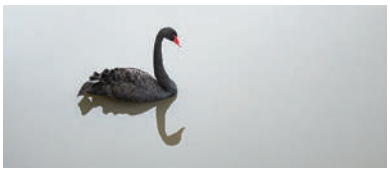
Probability Law 2

A probability of one means the event must happen.

For example, if we toss a coin, the probability of getting either a head or tail is one.

Probability Law 3

All probabilities must be between zero and one, inclusively. That is, $0 \le P(A) \le 1$.



Black Swan Events

Black Swan events are unexpected extreme events. The term stems from 16th century London: at this time, all known swans in the Euro-centric world were white. Subsequently, upon colonization of Western Australia, black swans were unexpectedly discovered. The term was popularized in Nassim Nicholas Taleb's book *The Black Swan: The Impact of the Highly Improbable* (2007). While the discovery of black swans did not adversely impact society, the black swan term today carries the connotations that the event is damaging, unexpected, and in hindsight, quite explainable. Two events occurring since 2000 that arguably qualify for black swan status are 9/11 and the disappearance of the MH-370 aircraft.

Statisticians quantify how rare events are via return periods. For example, if a 50-year earthquake at a fixed location has Richter magnitude 7.0, then the probability that a Richter magnitude 7.0 or greater earthquake occurs at the location over one year is roughly 1 / 50. Statisticians have a sub-discipline called extreme value theory that contains justifiable methods to estimate return periods (see Coles, 2001; *An Introduction to Statistical Modeling of Extreme Values*). This said, the field is often controversial and data void. Imagine trying to estimate a 200 year earthquake from only 50 years of data — a 200-year earthquake event is probably not contained in the data record!

While extreme value statisticians seldom refer to black swan events, the term is common in financial and insurance settings today. There, it often simply serves as a reminder that unexpected rare events do happen and are difficult to quantify.

Courtesy of Robert Lund