Procedure (cont.)

Wilcoxon Rank-Sum Test

If $n_1 > 10$, and the smaller sample size, n_1, is associated with Population X, then

$$z = \frac{T - \frac{n_1(n_1 + n_2 + 1)}{2}}{\sqrt{\frac{n_1 n_2 (n_1 + n_2 + 1)}{12}}}$$, where T is defined just as when $n_1 \le 10$.

Critical Value(s):

If $n_1 \le 10$:

If H_a is > One-Tailed: then reject H_0 if $T \ge T_U$, the critical value in Table K.

If H_a is < One-Tailed: then reject H_0 if $T \le T_L$, the critical value in Table K.

If H_a is ≠ Two-Tailed: then reject H_0 if $T \le T_L$, or $T \ge T_U$ the critical values in Table K.

If $n_1 > 10$:

if H_a is > One-Tailed, then reject H_0 if $z \ge z_\alpha$.

if H_a is < One-Tailed, then reject H_0 if $z \le -z_\alpha$.

if H_a is ≠ Two-Tailed, then reject H_0 if $z \le -z_{\alpha/2}$ or if $z \ge z_{\alpha/2}$.

Assumptions:

The data are such that they can be ranked. The two samples are selected in an independent and random fashion.

17.3 Exercises

Basic Concepts

1. What type of data is the Wilcoxon rank-sum test used to analyze?
2. What is the parametric test used to analyze the type of data that can also be analyzed by the Wilcoxon rank-sum test? What assumptions are associated with this test, and why are they sometimes not reasonable?
3. What assumptions are required for the Wilcoxon rank-sum test?
4. What levels of measurement may data possess in order for the Wilcoxon rank-sum test to be performed?
5. Describe the procedure for ranking data in order to perform a Wilcoxon rank-sum test.
6. What are the null and alternative hypotheses associated with the Wilcoxon rank-sum test?
7. What is the test statistic associated with the Wilcoxon rank-sum test for small samples? What does the test statistic depend on and how small is a *small* sample?
8. What is the test statistic associated with the Wilcoxon rank-sum test for large samples?
9. Identify the critical values associated with the Wilcoxon rank-sum test for both large and small samples.

Exercises

10. A luxury car dealer is considering two possible locations for a new auto mall. The rent on the south side of town is cheaper. However, the dealer believes that the average household income is significantly higher on the north side of town. The dealer has decided that he will locate the new auto mall on the north side of town if the results of a study which he has commissioned show that the median household income is significantly higher on the north side of town. The results of the study are as follows.

Household Incomes	
North Side ($)	South Side ($)
50,000	43,000
45,000	45,000
55,000	42,000
25,000	50,000
75,000	36,000
35,000	48,000
65,000	38,000
55,000	43,000
45,000	43,000

a. Use the Wilcoxon rank-sum test to determine if the auto dealer should locate the new auto mall on the north side of town. Use $\alpha = 0.05$.

b. What assumptions were made in performing the hypothesis test in part **a.**?

11. An internal auditor for Tiger Enterprises has been asked to determine if there is a difference in the amount charged for daily expenses by two top salesmen, Mr. Ellis and Mr. Ford. The auditor randomly selects seven days and determines the daily expenses for each of the salesmen.

Daily Expenses	
Mr. Ellis ($)	Mr. Ford ($)
55	60
53	55
58	65
54	50
56	70
55	55
55	65

a. Using the Wilcoxon rank-sum test, can the auditor conclude that there is a difference in the median amount charged for daily expenses by the two top salesmen, Mr. Ellis and Mr. Ford? Use $\alpha = 0.05$.

b. What assumptions were made in performing the test in part **a.**?

12. The Armed Forces have two different programs for training aircraft personnel. A government regulatory agency has been commissioned to evaluate any differences which may exist between the two programs. The agency administers a standardized test to randomly selected groups of students from the two programs. The results of the test for the students in each of the programs are as follows.

Standardized Test Scores

Program A	Program B
85	87
95	96
75	78
100	100
70	74
90	92
80	82

a. Using the Wilcoxon rank-sum test, can the agency conclude that there is a difference in the median test scores of students in the two programs? Use $\alpha = 0.10$.

b. What assumptions were made in performing the test in part **a.**?

13. Tom Anderson, a supply clerk with the Navy, has been asked to determine if a new battery which has been offered to the Navy (at a reduced price) has a shorter life than the battery which they are currently using. He randomly selects batteries of each type and allows them to run continuously so that he can measure the time until failure for each battery. The results of the test are as follows.

Time Until Failure for Batteries (Hours)

New Battery	Old Battery
655	745
730	675
670	730
715	690
685	760
745	660

a. Using the Wilcoxon rank-sum test, do the data suggest at $\alpha = 0.05$ that the median time until failure for the new battery is significantly less than the median time until failure for the old battery?

b. What assumptions were made in performing the test in part **a.**?

14. A cereal manufacturer has advertised that its product, Fiber Oat Flakes, has a lower fat content than its competitor, Bran Flakes Plus. Because of the complaints from the manufacturer of Bran Flakes Plus, the FDA has decided to test the claim that Fiber Oat Flakes has a lower median fat content than Bran Flakes Plus. Several boxes of each cereal are selected and the fat content per serving is measured. The results of the study are as follows.

Fat Content of Cereals (Grams)

Fiber Oat Flakes	Bran Flakes Plus
5	6
6	8
4	4
7	9
3	3
5	7
5	5
6	8
4	4

a. Using the Wilcoxon rank-sum test, does the study performed by the FDA substantiate the claim made by the manufacturer of Fiber Oat Flakes at $\alpha = 0.05$?

b. What assumptions were made in performing the test in part **a.**?

15. A Hollywood studio believes that a movie which is considered a drama will draw a larger crowd on average than a movie which is a comedy. To test this theory, the studio randomly selects several movies which are classified as dramas and several movies which are classified as comedies and determines the box office revenue for each movie. The results of the survey are as follows.

Box Office Revenues (Millions of Dollars)

Drama	Comedy
180	150
240	190
120	110
220	170
140	130

a. Using the Wilcoxon rank-sum test, do the data substantiate the studio's belief that dramas will draw a larger crowd on average than comedies at $\alpha = 0.05$?

b. What assumptions were made in performing the test in part **a.**?

16. *Consumer Magazine* is reviewing the top selling amplifiers produced by two major stereo manufacturers. One of the most important qualities of the amplifiers is the maximum power output. Brand A has redone their internal design and claims to have a higher maximum power level than Brand B. To test this claim, *Consumer Magazine* randomly selects amplifiers from each brand and determines the maximum power output. The results of the test are as follows.

Maximum Power Output (Watts)	
Brand A	Brand B
800	780
828	805
772	755
830	807
770	753
826	803
774	757

a. Using the Wilcoxon rank-sum test, do the data substantiate the claim that the Brand A amplifier has a higher median maximum power output than Brand B at $\alpha = 0.05$?

b. What assumptions were made in performing the test in part **a.**?

17. A state environmental board wants to compare pollution levels in two of its major cities. Sunshine City thrives on the tourist industry and Service City thrives on the service industry. The environmental board randomly selects several areas within the cities and measures the pollution levels in parts per million with the following results.

Pollution Levels (ppm)	
Sunshine City	Service City
8.50	7.90
9.00	8.35
8.00	7.45
9.07	8.40
7.93	7.40
9.14	8.45
7.86	7.35
8.50	7.90

a. Using the Wilcoxon rank-sum test, can the state environmental board conclude at $\alpha = 0.05$ that Service City has a lower pollution level on average than Sunshine City?

b. What assumptions were made in performing the test in part **a.**?

17.4 The Rank Correlation Test

In Section 4.7 we studied the correlation coefficient as a measure of association between two random variables. The parametric correlation coefficient gives a direct correlation between the variables. In this section we will transform the data from two variables into ranks and develop a method for detecting an association between them.