Since X = the number of times the less frequent sign occurs, $X = 15$, and the calculated value of the test statistic is

$$z = \frac{15 + 0.5 - \left(\frac{50}{2}\right)}{\frac{\sqrt{50}}{2}} \approx -2.69 .$$

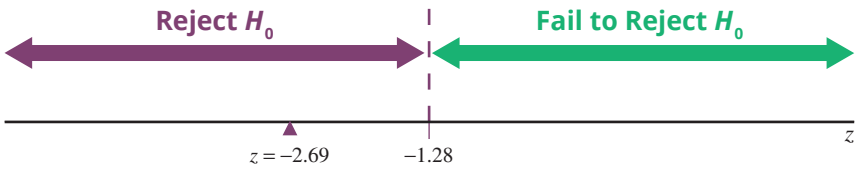

Figure 17.1.4

Step 6: Make the decision and state the conclusion in terms of the original question.

As shown in Figure 17.1.4, the value of the test statistic falls in the rejection region (-2.69 is less than -1.28). It is unlikely that the difference between the observed value and the hypothesized value is due to ordinary sampling variation. Thus, we reject the null hypothesis at $\alpha = 0.10$.

Conclusion and Interpretation: There is sufficient evidence for the Montgomery County Hospital CEO to conclude at $\alpha = 0.10$ that the median length of stay for her patients is significantly shorter than the median length of stay for the United States as a whole.

17.1 Exercises

Basic Concepts

1. What are parametric statistics?
2. Identify and explain the main disadvantage of the sign test.
3. Under what conditions are parametric statistical methods not appropriate for data analysis?
4. Identify the three characteristics of nonparametric statistical methods.
5. What are the disadvantages of nonparametric statistics?
6. The sign test and the Wilcoxon signed-rank test are designed to conduct hypothesis tests involving which kind(s) of experiments? What is the corresponding parametric statistical technique used to analyze these types of experiments?
7. What assumptions are made when conducting the sign test?
8. Name the two ways that the sign test can be used to perform hypothesis tests.
9. How do the rejection regions for nonparametric tests differ from those for parametric tests? Explain.
10. What is done with measurements that have a difference of zero in a paired difference experiment? Why is this the case?
11. What are the null and alternative hypotheses associated with the sign test?
12. What is the test statistic for the sign test for small samples? How small is a *small* sample?
13. What is the test statistic for the sign test for large samples? How large is a *large* sample?

14. Identify the critical values and rejection rules for both small and large samples with regard to the sign test.

Exercises

15. Hurricane Hugo swept through the Lowcountry in South Carolina causing billions of dollars of damage. In the past, the median claim for homes damaged by hurricanes for an insurance company in the Lowcountry had been $25,000. The insurance company believes that the median claim will be significantly larger for homes damaged by Hugo than past hurricanes. In order to investigate this theory, the insurance company randomly selects 55 homes and sends adjusters to settle the claims. In the sample of 55 homes, 40 of the homes had a claim in excess of the historical median. Is there overwhelming evidence at $\alpha = 0.10$ that the median claim for home damage from Hurricane Hugo was greater than the historical median?

16. The manufacturer of Brand X floor polish is developing a new polish that they hope will dry faster than the competition's polish. The competition's polish is advertised to have an average (median) drying time of 10 minutes. In a random sample of 1000 polishes with the new polish, 700 of the polishes dried in less than 10 minutes. Based on the data, can the manufacturer conclude that the median drying time for Brand X is faster than the competition's brand at a 0.05 level of significance?

17. NarStor, a computer disk drive manufacturer, claims that the median time until failure for their hard drives is 14,400 hours. You work for a consumer group that has decided to examine this claim. Technicians ran 16 NarStor hard drives continuously for almost three years. Recently the last drive failed. The times to failure (in hours) are given in the following table.

Time Until Hard Drive Failure (Hours)							
330	620	1870	2410	4620	6396	7822	8102
8309	12,882	14,419	16,092	18,384	20,916	23,812	25,814

a. Is there overwhelming evidence that the median time until failure is less than the manufacturer claims? Use $\alpha = 0.10$.

b. What assumption did you make in performing the test in part **a.**?

18. A.C. Bone has developed a duck hunting boot which it claims can remain immersed for more than 12 hours without leaking. 15 of the boots are tested and the time until first leakage is measured. Nine of the boots last more than 12 hours without leaking.

a. Do the data substantiate A.C. Bone's claim at $\alpha = 0.05$?

b. What assumption did you make in performing the test in part **a.**?

19. Given that most textbooks can now be purchased online, one wonders if students can save money by comparison shopping for textbooks at online retailers and at their local bookstores. To investigate, students at Tech University randomly sampled 25 textbooks on the shelves of their local bookstores. The students then found the "best" available price for the same textbooks via online retailers. The prices for the textbooks are listed in the following table.

Textbook Prices

	Price ($)			Price ($)			Price ($)	
Textbook	Bookstore	Online Retailer	Textbook	Bookstore	Online Retailer	Textbook	Bookstore	Online Retailer
1	70	60	10	97	86	19	49	40
2	38	36	11	140	130	20	149	127
3	88	89	12	40	30	21	126	130
4	165	149	13	175	150	22	92	93
5	80	136	14	85	75	23	144	129
6	103	95	15	100	85	24	98	84
7	42	50	16	68	62	25	40	52
8	98	111	17	67	69			
9	89	65	18	140	142			

Using the data in the table, and without making any distributional assumptions, is it less expensive for the students to purchase textbooks from the online retailers than the local bookstores? Use $\alpha = 0.01$.

20. The management for a large grocery store chain would like to determine if a new cash register will enable cashiers to process a larger number of items on average than the cash register which they are currently using. Seven cashiers are randomly selected, and the number of grocery items which they can process in three minutes is measured for both the old cash register and the new cash register. The results of the test are as follows.

Number of Grocery Items Processed in Three Minutes

Cashier	1	2	3	4	5	6	7
Old Register	60	70	55	75	62	52	58
New Register	65	71	55	75	65	57	57

Without making any assumptions about the distribution, can management conclude that the new cash register will allow cashiers to process a significantly larger number of items on average than the old cash register at $\alpha = 0.05$.

21. An auto dealer is marketing two different models of a high-end sedan. Since customers are particularly interested in the safety features of the sedans, the dealer would like to determine if there is a difference in the braking distance (the number of feet required to go from 60 mph to 0 mph) of the two sedans. Six drivers are randomly selected and asked to participate in a test to measure the braking distance for both models. Each driver is asked to drive both models and brake once they have reached exactly 60 mph. The distance required to come to a complete halt is then measured in feet. The results of the test are as follows.

Braking Distance of High-End Sedans (in Feet)

Driver	1	2	3	4	5	6
Model A	150	145	160	155	152	153
Model B	152	146	160	157	154	155

Without making assumptions about the distribution of the data, can the auto dealer conclude that there is a significant difference in the braking distance of the two models of high end sedans? Use $\alpha = 0.10$.

22. A nutritionist is interested in determining the decrease in cholesterol level which a person can achieve by following a particular diet which is low in fat and high in fiber. Seven subjects are randomly selected to try the diet for six months, and their cholesterol levels are measured both before and after the diet. The results of the study are as follows.

Cholesterol Levels							
Subject	1	2	3	4	5	6	7
Before Diet	155	170	145	200	162	180	160
After Diet	152	168	148	195	162	178	157

Can the nutritionist conclude that there is a significant decrease in average cholesterol level when the diet is used? We don't have any knowledge about the distribution of the data. Use $\alpha = 0.01$.

17.2 The Wilcoxon Signed-Rank Test

A disadvantage of the sign test is that it wastes information. The sign test merely counts the number of positive or negative signs in a paired difference experiment and ignores the magnitude of the differences. The **Wilcoxon signed-rank test** is a nonparametric technique which can also be used to evaluate a paired difference experiment. This test is designed to detect populations whose centers are shifted to the right or the left of each other. As with the sign test, no distributional assumption is required. However, the pairs of data must have been randomly selected, and it must be possible to rank the differences.

An advantage of the Wilcoxon signed-rank test is that it does not ignore the magnitudes of the differences. However, it does not take the magnitude directly into account. Instead, the ranks of the data are analyzed.

Ranking is nothing new. It simply requires putting the data in order from smallest to largest and attaching a rank to each data item. In general, the lowest value is assigned a rank of one and the highest value is assigned a rank of n, where n is the number of nonzero differences. How do we handle ties? If there are two or more values with the same magnitude, these values will each be assigned the same rank, which is equal to the average of the ranks which would have been assigned to these values if they had slightly different consecutive values. The ranking procedure is explained more fully in the following example.

Example 17.2.1

Ranking Quantitative Data

Rank the following stocks, traded on the New York Stock Exchange, from smallest price to largest price.

Table 17.2.1 – Stock Prices	
Stock	**Price per Share ($)**
Merck & Co., Inc.	78.25
AT&T, Inc.	27.15
SecureWorks Corp.	28.04
Micron Technology, Inc.	73.21
HP Inc.	25.16
AMC Entertainment Holdings, Inc.	30.50
CitiGroup, Inc.	60.12
Exxon Mobil Co.	55.30
AutoCanada Inc.	30.50