$$H_0: \frac{\sigma^2_{Max}}{\sigma^2_{Min}} = 1$$

against an alternative of

$$H_a: \frac{\sigma^2_{Max}}{\sigma^2_{Min}} \neq 1.$$

where σ^2_{Max} represents the largest variance of the three age groups and σ^2_{Min} represents the smallest variance of the three age groups. The rationale is that if there is not a significant difference between the largest and smallest variances, then there won't be a significant difference among all group variances.

Of course, if there is a significant difference between the largest and smallest variances, then our assumption is violated, and we may need to do one of the following: transform the data by taking the natural log or square root of the responses; use nonparametric procedures; or use some alternative statistics such as Welch's or Brown-Forsythe procedures which use an alternative F-statistic to determine if you have statistical significance.

Understanding that the data are collected from three independent normally distributed populations and that we are testing the ratio of two variances, the test statistic is given by

$$F = \frac{s^2_{Max}}{s^2_{Min}}.$$

Suppose we are testing at the 5% level of significance (i.e., $\alpha = 0.05$). Additionally, we know that we have a two-sided test based on the alternative hypothesis. Since the alternative hypothesis is two-sided, two tails of the F-distribution must be determined as rejection regions. That is, we want to determine if $F \le F_{1-\alpha/2, df_{\text{num}}, df_{\text{den}}}$ or if $F \le F_{\alpha/2, df_{\text{num}}, df_{\text{den}}}$. These critical values are

$$F_{1-\alpha/2, df_{\text{num}}, df_{\text{den}}} = F_{0.975,49,49} = 0.5675$$
$$F_{\alpha/2, df_{\text{num}}, df_{\text{den}}} = F_{0.025,49,49} = 1.7622$$

The rejection region is that we will reject the null hypothesis if the F-test statistic is less than or equal to 0.5675 or if the F-test statistic is greater than or equal to 1.7622.

As can be seen in the JMP output, the standard deviation (and thus, the variance) is largest for the group of teens (13–18 years old) and the standard deviation is smallest for the adults (more than 18 years old).

The test statistic is given by

$$F = \frac{s^2_{Max}}{s^2_{Min}} = \frac{(25.5704)^2}{(20.3235)^2} \approx 1.5830.$$

Since the test statistic does not fall in the rejection region, we fail to reject the null hypothesis and conclude that there is no evidence to indicate that the variances of screen time among tweens, teens, and adults are significantly different.

Technology

Using the Excel function, F.INV.RT, we can find the critical value $F_{0.975,49,49}$ by typing the following into a cell of the spreadsheet "=F.INV.RT(0.975, 49, 49)" which equals 0.5675.

For instructions on finding *F* critical values using technology visit stat.hawkeslearning.com and navigate to **Discovering Business Statistics, Second Edition > Technology Instructions > *F*-Distribution > Critical Value**.

12.2 Exercises

Basic Concepts

1. Why is it important to validate the assumptions upon which a hypothesis test is based?
2. What is the first assumption on which ANOVA is based?
3. How can we test to see if the data reasonably satisfy the first assumption?
4. What is the second assumption on which ANOVA is based?

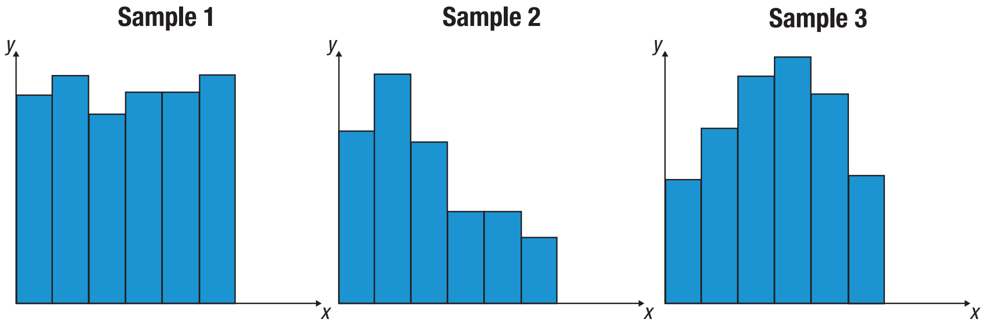
5. How can we determine if the second assumption is reasonable for the data we are interested in?

6. What is a simple "rule of thumb" that may be used to check the second assumption?

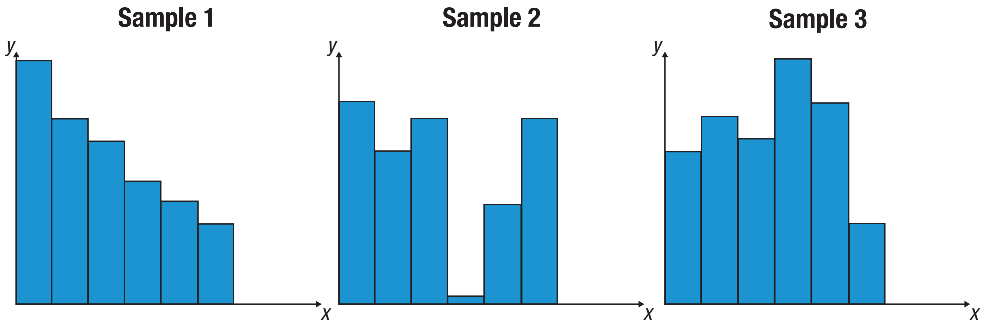
7. What is the third assumption that must be met before performing ANOVA?

Exercises

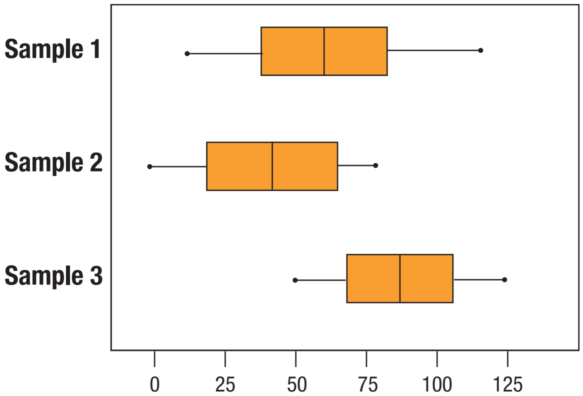
8. For each of the following histograms of sample data, decide whether or not you think it is reasonable to assume that the data were drawn from a population that has an approximately normal distribution.



9. For each of the following histograms of sample data, decide whether or not you think it is reasonable to assume that the data were drawn from a population that has an approximately normal distribution.

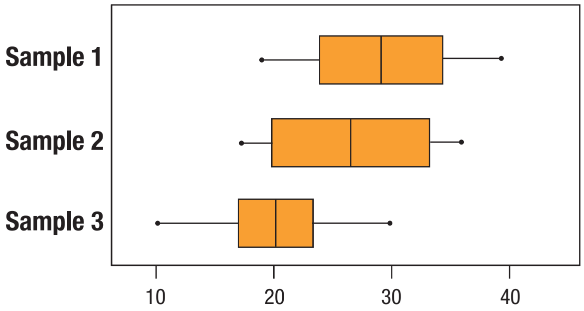


10. Consider the following box plots.



Do you think it is reasonable to assume that the three populations represented by the sample data in these box plots have equal variances? Explain.

11. Consider the following box plots.



Do you think it is reasonable to assume that the three populations represented by the sample data in these box plots have equal variances? Explain.

12. Consider the following data on the diameter measurements (in inches) of soft drink bottles of three different brands.

Pepsi	Coca-Cola	Dr. Pepper
1.17	0.14	2.17
1.20	0.07	2.20
1.15	0.11	2.16
1.21	0.08	2.20
1.07	0.21	2.31
1.18	0.22	2.09
1.12	0.15	2.24
1.09	0.08	2.07
1.30	0.12	2.05
1.11	0.28	2.18

Test the assumption of homogeneity required to conduct the ANOVA test, that is, the three samples come from populations with equal variances at a 0.05 significance level.

13. Consider the given data on the foot lengths (in cms) of adult males obtained from three different states of the U.S.

Iowa	Hawaii	California
177.23	172.24	170.28
175.79	172.20	177.46
176.76	176.46	174.67
180.52	180.23	186.57
185.28	171.68	189.29
179.62	177.98	178.63
188.47	179.68	179.91
179.63	177.00	184.72
175.24	179.25	171.52
180.81	181.97	179.08
178.50	182.45	185.49
190.01	175.97	180.24
188.36	176.84	177.64
182.49	169.96	176.85
180.73	167.56	178.72

Do the data satisfy the assumption that the samples come from normally distributed populations? (Use the Shapiro-Wilk test or the Anderson-Darling test.)

14. Four different samples of 10 students each from the same age group are given four types of riddles to solve every week for a period of two months.

    The standard deviations of the time to study and solve the riddles for each of the four samples of students are given below.

	Sample 1	Sample 2	Sample 3	Sample 4
Standard deviation (in hours/day)	2.24	1.10	5.49	2.72

    Using the "rule of thumb" for equal variances, examine whether the four samples of study hours can be considered to come from populations with the same variance.

15. A survey is conducted among three different age groups of 20 people each at a shopping center located near the researcher's residence. The three samples are from the following age groups: Teen (15-19 years), Young Adult (20-30 years), and Adult (31 years and above). The researcher asks the participants about the amount of money they spend monthly on shopping.

    A one-way ANOVA is conducted to test the difference between the mean amounts spent by the respondents for the three different age groups. Consider the assumptions necessary to perform an ANOVA and identify which assumption, if any, is violated in this scenario.

16. An assumption of the ANOVA test is that the k samples considered should come from populations with the same variance. If a boxplot is created for each sample on the same scale, what characteristic should be examined to conclude that the populations have the same variance?

17. Three samples of ten college students were randomly selected from the Japanese club, the Computer Science club, and the soccer team. Each sample was surveyed on their political views of the president. Can the samples in this scenario be considered independent? Explain your answer.

12.3 The *F*-Distribution and the *F*-Test

In Section 12.1, we introduced most of the formulas that are used to analyze data from more than two samples in order to determine whether or not there is a significant difference among population means. We developed MST, a measure for summarizing the variability among the sample means, and MSE, a measure for summarizing the variability within the samples themselves. We determined that if the variability among the sample means is much larger than the variability within the sample observations, we will doubt the hypothesis that the population means are the same. Alternatively, if the variability among the sample means is small when compared to the variability within the sample observations, it is not likely that the population means are significantly different.

Consider the ratio of the MST (mean square for treatments), the summary measure of the variability among the sample means, to the MSE (mean square for error), the summary measure of the variability within the samples.

$$\frac{\text{MST}}{\text{MSE}}$$

The ***F*-distribution**, named after the English statistician Sir Ronald Fisher, is a continuous distribution. It will be used in this and subsequent chapters to analyze variation in test statistics formed as ratios of two random variables. The *F*-distribution is not symmetrical; rather, it is skewed to the right. Like the *t*-distribution, its parameters are degrees of freedom. *F*-distributions are associated with test statistics that are quotients. What distinguishes the *F*-distribution is that it has a pair of values for its degrees of freedom. The number of degrees