Step 5: Collect sample data and compute the value of the test statistic.

The test statistic is given by

$$F = \frac{s_1^2}{s_2^2} = \frac{2500}{900} = 2.7778.$$

This statistic indicates that the variance of revenue for dramas is close to three times that of the variance of the revenue of comedies. Does that indicate that the ratio of the variances is significantly different at the 5% level? We will discuss in **Step 6**.

Technology

To find the P-value for an F-distribution using technology, please visit stat.hawkeslearning.com and navigate to **Discovering Business Statistics, Second Edition > Technology Instructions > *F*-Distribution > *F*-Probability (cdf)**.

Step 6: Make the decision and state the conclusion in terms of the original question.

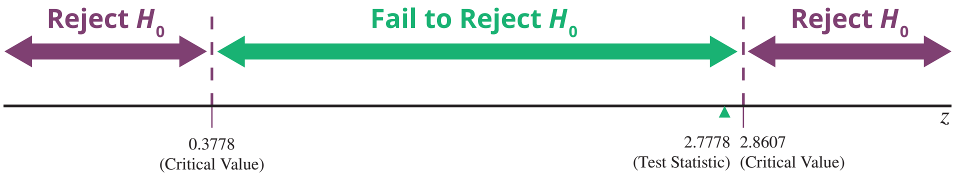

Figure 11.5.7

Since the test statistic does not fall in the rejection region, we fail to reject the null hypothesis.

Using the P-value approach, we want to find $2P(F > 2.7778)$ since this is a two-tailed test. This results in a P-value of 0.0564. Since the P-value is greater than the significance level of 0.05, we fail to reject the null hypothesis.

Conclusion and Interpretation: We conclude that there is insufficient evidence to indicate that the variances in revenue between dramas and comedies are significantly different.

Please note that the methods presented in this section work very poorly when the normality assumption is violated. It is very important to validate the assumption of normality before developing confidence intervals or performing hypothesis tests on the ratio of the variances.

11.5 Exercises

Basic Concepts

1. Give two examples of situations in which someone would be interested in comparing population variances (or standard deviations).
2. What assumptions are necessary to perform a hypothesis test for two population variances?
3. What is the test statistic that is used when comparing two population variances?
4. What are the parameters of the distribution of the test statistic in the previous question?

Exercises

5. Find a point on the F-distribution with 7 numerator degrees of freedom and 22 denominator degrees of freedom such that the following area lies to the right of this value.

   a. $\alpha = 0.100$

   b. $\alpha = 0.050$

   c. $\alpha = 0.025$

   d. $\alpha = 0.010$

6. Find a point on the F-distribution with 30 numerator degrees of freedom and 8 denominator degrees of freedom such that the following area lies to the right of this value.

a. $\alpha = 0.100$

b. $\alpha = 0.050$

c. $\alpha = 0.025$

d. $\alpha = 0.010$

7. Find $F_{0.025}$ for an F-distribution with the following parameters.

a. 1 numerator degree of freedom, 25 denominator degrees of freedom

b. 6 numerator degrees of freedom, 11 denominator degrees of freedom

c. 8 numerator degrees of freedom, 40 denominator degrees of freedom

d. 3 numerator degrees of freedom, 18 denominator degrees of freedom

8. Find $F_{0.010}$ for an F-distribution with the following parameters.

a. 15 numerator degrees of freedom, 19 denominator degrees of freedom

b. 10 numerator degrees of freedom, 29 denominator degrees of freedom

c. 60 numerator degrees of freedom, 24 denominator degrees of freedom

d. 12 numerator degrees of freedom, 21 denominator degrees of freedom

9. State the null and alternative hypotheses for each scenario.

a. A professor believes that the variance of SAT scores of honor students is less than that of all students who take the SAT. Let σ_1^2 represent the population variance for honor students.

b. A quality control inspector believes that the variance in the diameters of soda cans produced by Machine 1 is greater than the variance in the diameters of soda cans produced by Machine 2. Let σ_1^2 represent the population variance for Machine 1.

10. Calculate the test statistic for a hypothesis test for two population variances using the given information. Assume that both population distributions are approximately normal.

$$n_1 = 4, \quad s_1^2 = 0.961, \quad n_2 = 6, \quad s_2^2 = 0.899$$

11. State the critical value(s) of the test statistic, and determine the rejection region for the hypothesis test for the two population variances using the given information. Then give the appropriate conclusion for the hypothesis test. Assume that both population distributions are approximately normal.

a. $n_1 = 14, \quad s_1^2 = 3.152, \quad n_2 = 11, \quad s_2^2 = 9.300, \quad H_a\colon \sigma_1^2 < \sigma_2^2, \quad \alpha = 0.05$

b. $n_1 = 12, \quad s_1^2 = 1893, \quad n_2 = 26, \quad s_2^2 = 1066, \quad H_a\colon \sigma_1^2 > \sigma_2^2, \quad \alpha = 0.01$

c. $n_1 = 20, \quad s_1^2 = 27.08, \quad n_2 = 29, \quad s_2^2 = 11.77, \quad H_a\colon \sigma_1^2 \neq \sigma_2^2, \quad \alpha = 0.05$

For exercises 12-16, complete the following steps. Assume that both population distributions are approximately normal in each scenario.

a. State the null and alternative hypotheses.

b. Determine which distribution to use for the test statistic and state the level of significance.

c. Calculate the test statistic.

d. Draw a conclusion and interpret the decision.

12. A golf pro believes that the variances of his driving distances are different for different brands of golf balls. In particular, he believes that his driving distances, measured in yards, have a smaller variance when he uses Titleist golf balls than when he uses a generic store brand. He hits 10 Titleist golf balls and records a sample variance of 201.65. He hits 10 generic golf balls and records a sample variance of 364.57. Test the golf pro's claim using a 0.05 level of significance. Assume the samples are from populations that are approximately normally distributed. Does the evidence support the golf pro's claim?

13. A quality control inspector believes that the variance in the diameters of soda cans, measured in millimeters, is greater for soda cans produced by Machine A than for soda cans produced by Machine B. The sample variance of a random sample of 15 soda cans from Machine A is 2.788. The sample variance for a random sample of 17 soda cans from Machine B is 1.982. Test the inspector's claim using a 0.10 level of significance. Assume the samples are from populations that are approximately normally distributed. Does the evidence support the inspector's claim?

14. A medical researcher believes that the variance of total cholesterol levels in men is greater than the variance of total cholesterol levels in women. The sample variance for a random sample of 8 men's cholesterol levels, measured in mg/dL, is 277. The sample variance for a random sample of 7 women is 89. Test the researcher's claim using a 0.10 level of significance. Assume the samples are from populations that are approximately normally distributed. Does the evidence support the researcher's belief?

15. A basketball coach believes that the variance of the heights of adult male basketball players is different from the variance of heights for the general population of men. The sample variance of heights, measured in inches, for a random sample of 12 basketball players is 24.76. The sample variance for a random sample of 13 other men is 25.87. Test the coach's claim using a 0.01 level of significance. Assume the samples are from populations that are approximately normally distributed. Does the evidence support the coach's claim?

16. One study claims that the variance in the resting heart rates of smokers is different than the variance in the resting heart rates of nonsmokers. A medical student decides to test this claim. The sample variance of resting heart rates, measured in beats per minute, for a random sample of 5 smokers is 545.1. The sample variance for a random sample of 5 nonsmokers is 103.7. Test the study's claim using a 0.01 level of significance. Assume the samples are from populations that are approximately normally distributed. Does the evidence support the study's claim?