Step 6: Make the decision and state the conclusion in terms of the original question.

As shown in Figure 11.3.3, the value of the test statistic falls in the rejection region to the left. The test statistic indicates that the observed average daily sales are more than 9 standard deviations below the hypothesized value of 0. It is highly unlikely that the difference between the observed value and the hypothesized value is due to ordinary sampling variation. Thus, the null hypothesis is rejected at $\alpha = 0.01$.

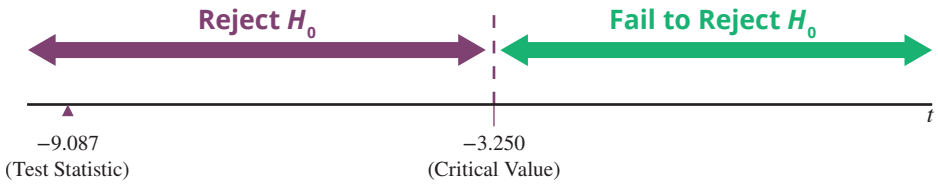

Figure 11.3.3

Technology

For technology instructions on finding the *P*-value for a given *t*-statistic, please visit stat.hawkeslearning.com and navigate to **Discovering Business Statistics, Second Edition > Technology Instructions > *t*-Distribution > *t*-Probability(cdf)**.

To find the *P*-value for this test, we use the same methodology as in Example 11.2.1. The *P*-value is given by

$$P\text{-value} = 2P(t < |-9.087|) = 2P(t > 9.087).$$

We cannot use the *t*-table to find $P(t > 9.087)$; we can only put bounds on the probability. Thus, at 9 degrees of freedom, the largest value in the respective row is 3.250. The best information that we can gain from the *t*-table is that $P(t > 9.087) < 0.005$. Since we need to multiply the probability by 2, we have $P\text{-value} < 0.01$.

Since we are performing the test at $\alpha = 0.01$, we reject the null hypothesis because the *P*-value is less than α—the same decision we made using the rejection region approach.

Conclusion and Interpretation: There is sufficient evidence for the owner to conclude at the $\alpha = 0.01$ level that the average daily sales between the two restaurants are significantly different.

11.3 Exercises

Basic Concepts

1. Describe the differences between an independent experimental design and a paired design.
2. What are the assumptions for a paired difference experimental design?
3. What is the appropriate statistical measure to use when performing a hypothesis test about a paired difference experiment?
4. How does the hypothesis testing procedure for a paired difference experiment differ from that of a two-sample *t*-test?
5. What is the test statistic used in a paired difference hypothesis test?

Exercises

6. Determine the critical value(s) of the test statistic for each of the following paired difference tests (assume the differences have an approximately normal distribution).

a. Left-tailed test, $\alpha = 0.01$, $n_d = 15$

b. Right-tailed test, $\alpha = 0.10$, $n_d = 20$

c. Two-tailed test, $\alpha = 0.05$, $n_d = 8$

7. Determine the critical value(s) of the test statistic for each of the following paired difference tests (assume the differences have an approximately normal distribution).

a. Left-tailed test, $\alpha = 0.005$, $n_d = 12$

b. Right-tailed test, $\alpha = 0.025$, $n_d = 5$

c. Two-tailed test, $\alpha = 0.10$, $n_d = 25$

8. Given that most textbooks can now be purchased online, one wonders if students can save money by comparison shopping for textbooks at online retailers and at their local bookstores. To investigate, students at Tech University randomly sampled 25 textbooks on the shelves of their local bookstores. The students then found the "best" available price for the same textbooks via online retailers. The prices for the textbooks are listed in the following table.

Textbook Prices

	Price ($)			Price ($)	
Textbook	Bookstore	Online Retailer	Textbook	Bookstore	Online Retailer
1	70	60	14	85	75
2	38	36	15	100	85
3	88	89	16	68	62
4	165	149	17	67	69
5	80	136	18	140	142
6	103	95	19	49	40
7	42	50	20	149	127
8	98	111	21	126	130
9	89	65	22	92	93
10	97	86	23	144	129
11	140	130	24	98	84
12	40	30	25	40	52
13	175	150			

Data

This data set can be found on stat.hawkeslearning.com under **Discovering Business Statistics, Second Edition > Data Sets > Textbook Prices**.

a. Is a paired design appropriate for the above study? Explain.

b. What assumption must be made in order to perform the test of hypothesis?

c. Do the data appear to satisfy the assumption described in part **b.**? Why or why not?

d. Based on the data, is it less expensive for the students to purchase textbooks from the online retailers than from local bookstores? Use $\alpha = 0.01$.

e. Calculate a 99% confidence interval for the mean difference in cost between the bookstores and the online retailers. Interpret the interval.

9. The management for a large grocery store chain would like to determine if a new cash register will enable cashiers to process a larger number of items on average than the cash register they are currently using. Seven cashiers are randomly selected, and the number of grocery items they can process in three minutes is measured for both the old cash register and the new cash register. The results of the test are as follows.

Number of Grocery Items Processed in Three Minutes							
Cashier	1	2	3	4	5	6	7
Old Cash Register	60	70	55	75	62	52	58
New Cash Register	65	71	55	75	65	57	57

a. Is a paired design appropriate for the above experiment? Explain.

b. What assumption must be made in order to perform the test of hypothesis?

c. Do the data appear to satisfy the assumption described in part **b.**? Why or why not?

d. Calculate a 95% confidence interval for the mean difference between the number of items processed using the old cash register and the new cash register. Interpret this interval.

e. Can the management conclude that the new cash register will allow cashiers to process a significantly larger number of items on average than the old cash register at $\alpha = 0.05$?

10. An auto dealer is marketing two different models of a high-end sedan. Since customers are particularly interested in the safety features of the sedans, the dealer would like to determine if there is a difference in the braking distance (the number of feet required to go from 60 mph to 0 mph) of the two sedans. Six drivers are randomly selected and asked to participate in a test to measure the braking distance for both models. Each driver is asked to drive both models and brake once they have reached exactly 60 mph. The distance required to come to a complete halt is then measured in feet. The results of the test are as follows.

Braking Distance of High-End Sedans (Feet)						
Driver	1	2	3	4	5	6
Model A	150	145	160	155	152	153
Model B	152	146	160	157	154	155

a. Is a paired design appropriate for the above experiment? Explain.

b. What assumption must be made in order to perform the test of hypothesis?

c. Do the data appear to satisfy the assumption described in part **b.**? Why or why not?

d. Calculate a 90% confidence interval for the average difference between braking distances for Model A and Model B. Interpret the interval.

e. Can the auto dealer conclude that there is a significant difference in the braking distances of the two models of high-end sedans? Use $\alpha = 0.10$.