Step 4: Determine the critical value of the test statistic.

Since we have a two-tailed alternative hypothesis and a significance level of 5% (i.e., $\alpha = 0.05$), the critical value will be $t_{\alpha/2,df}$ where the degrees of freedom is calculated as follows.

$$df = \frac{\left(\frac{15.57}{10} + \frac{3.07}{10}\right)^2}{\frac{1}{10-1}\left(\frac{15.57}{10}\right)^2 + \frac{1}{10-1}\left(\frac{3.07}{10}\right)^2} \approx 12.42 .$$

We will use 12 degrees of freedom for this t-test which yields a critical value of $t_{0.05/2,12} = t_{0.025,12} = 2.179$. We will reject the null hypothesis if the test statistic is greater than or equal to 2.179 or if the test statistic is less than or equal to –2.179.

Step 5: Collect the sample data and compute the value of the test statistic.

Using the values in the table above, the test statistic is

$$t = \frac{(27.7 - 22.8) - 0}{\sqrt{\frac{15.57}{10} + \frac{3.07}{10}}} \approx 3.59 .$$

Step 6: Make the decision and state the conclusion in terms of the original question.

The critical values of the test statistic are ±2.179. Thus, since the test statistic, t, is greater than 2.179 we reject the null hypothesis in favor of the alternative.

Using the Technology Instructions for conducting the Two-Sample t-Test, we get P-value = 0.0035. Since we are performing the test using a significance level of 0.05, the P-value approach still leads us to reject the null hypothesis in favor of the alternative.

Conclusion and Interpretation: This indicates that there is evidence to conclude that the average miles per gallon between the Porsche Cayenne Hybrid model and the Porsche Cayenne Gas model are significantly different.

Note that we did not test whether the average miles per gallon for one model was more or less than the other, thus, the conclusion remains consistent with the stated hypotheses.

Technology

For technology instructions to do a two-sample hypothesis test of the means using the t-distribution, visit stat.hawkeslearning.com and navigate to **Discovering Business Statistics, Second Edition > Technology Instructions > Hypothesis Testing > Two-Sample *t*-Test**.

11.2 Exercises

Basic Concepts

1. Why might large samples not be available when attempting to make inferences about two population means?
2. What assumptions are necessary to perform a test for the difference between two population means when the population variances are unknown?
3. What is the test statistic for an hypothesis test about two population means when the population variances are unknown? How does this statistic differ from the test statistic used in Section 11.1?
4. What is a pooled variance? Why is it used?

Exercises

5. Determine the critical value(s) of the test statistic for each of the following tests for the comparison of two population means where the assumptions of normality and equal variance have been satisfied.

a. Left-tailed test, $\alpha = 0.05$, $n_1 = 10$, $n_2 = 15$

b. Right-tailed test, $\alpha = 0.10$, $n_1 = 8$, $n_2 = 12$

c. Two-tailed test, $\alpha = 0.01$, $n_1 = 5$, $n_2 = 7$

6. Determine the critical value(s) of the test statistic for each of the following tests for the comparison of two population means where the assumptions of normality and equal variance have been satisfied.

a. Left-tailed test, $\alpha = 0.025$, $n_1 = 13$, $n_2 = 25$

b. Right-tailed test, $\alpha = 0.005$, $n_1 = 7$, $n_2 = 18$

c. Two-tailed test, $\alpha = 0.10$, $n_1 = 15$, $n_2 = 15$

7. *Popular Science* (Vol. 242, No. 3) reported the results of a comparison of several popular minivans. One of the features that they compared was the time required to accelerate from 0 to 60 miles per hour in seconds. The Dodge Grand Caravan ES was able to accelerate from 0 to 60 mph in 11.3 seconds, on average. The Volkswagen Eurovan took 16.5 seconds on average to accelerate from 0 to 60 mph. Suppose that 15 minivans of each type were tested and that the sample standard deviation of the times required to accelerate from 0 to 60 for each minivan was 4 seconds. Assume that the population variances are approximately equal.

a. Calculate a 95% confidence interval for the difference in average acceleration time between the two types of minivans. Interpret the interval.

b. Do the data suggest that there is a significant difference in the time required to accelerate from 0 to 60 between the two types of minivans at $\alpha = 0.05$?

c. What assumptions did you make about the time required to accelerate from 0 to 60 mph in calculating the confidence interval in part **a.** and for performing the test in part **b.**?

8. A cereal manufacturer has advertised that its product, Fiber Oat Flakes, has a lower fat content than its competitor, Bran Flakes Plus. Because of complaints from the manufacturers of Bran Flakes Plus, the FDA has decided to test the claim that Fiber Oat Flakes has a lower average fat content than Bran Flakes Plus. Several boxes of each cereal are selected and the fat content per serving is measured. The results of the study are as follows. Assume that the population variances are approximately equal.

Fat Content (Grams)			
	n	$\bar{x}$	s
Fiber Oat Flakes	16	5	1
Bran Flakes Plus	15	6	2

a. Calculate a 90% confidence interval for the difference in average fat content between Fiber Oat Flakes and Bran Flakes Plus. Interpret the interval.

b. Does the study performed by the FDA substantiate the claim made by the manufacturer of Fiber Oat Flakes at $\alpha = 0.10$?

c. What assumptions must be made in order to calculate the confidence interval in part **a.** and perform the hypothesis test in part **b.**?

9. A large construction company would like to expand its operations into a new geographic area. The company has narrowed the choice of locations down to two cities. A major consideration in deciding between the two cities will be the average hourly wage they must pay for general laborers. The company randomly selects laborers from each city and determines their hourly wage with the following results. Assume that the population variances are approximately equal.

Hourly Wages (Dollars)			
	n	$\bar{x}$	s
City A	20	\$7	\$3
City B	20	\$8	\$2

a. Calculate a 99% confidence interval for the difference in average hourly wage between City A and City B. Interpret the interval.

b. Do the data indicate that there is a significant difference in hourly wages at $\alpha = 0.05$?

c. Calculate the P-value for the test performed in part **b.**

d. What assumptions must be made in order to calculate the confidence interval in part **a.** and perform the hypothesis test in part **b.**?

10. A Hollywood studio believes that a movie that is considered a drama will draw a larger crowd on average than a movie that is a comedy. To test this theory, the studio randomly selects several movies that are classified as dramas and several movies that are classified as comedies and determines the box office revenue for each movie. The results of the survey are as follows. Assume that the population variances are approximately equal.

Box Office Revenues (Millions of Dollars)			
	n	$\bar{x}$	s
Drama	15	180	50
Comedy	13	150	30

a. Calculate a 95% confidence interval for the difference in average revenue at the box office for drama and comedy movies. Interpret the interval.

b. Do the data substantiate the studio's belief that dramas will draw a larger crowd on average than comedies at $\alpha = 0.01$?

c. Calculate the P-value for the test you conducted in part **b.**

d. What assumptions must be made in order to calculate the confidence interval in part **a.** and to perform the hypothesis test in part **b.**?

11. *Consumer Magazine* is reviewing the top of the line amplifiers produced by two major stereo manufacturers. One of the most important qualities of the amplifiers is the maximum power output. Brand A has redone their internal design and claims to have a higher maximum power level than Brand B. To test this claim, *Consumer Magazine* randomly selects amplifiers from each brand and determines the maximum power output. The results of the test are as follows. Assume that the population variances are approximately equal.

Amplifier Power Output (Watts)			
	n	$\bar{x}$	s
Brand A	12	800	25
Brand B	10	780	25

a. What assumptions must be made in order to perform the hypothesis test?

b. Do the data substantiate the claim that the Brand A amplifier has a higher average maximum power output than Brand B at $\alpha = 0.05$?

12. The State Environmental Board wants to compare pollution levels in two of its major cities. Sunshine City thrives on the tourist industry and Service City thrives on the service industry. The environmental board randomly selects several areas within the cities and measures the pollution levels in parts per million with the following results. Assume that the population variances are approximately equal.

Pollution Levels (ppm)			
	n	$\bar{x}$	s
Sunshine City	15	8.5	0.57
Service City	10	7.9	0.50

a. What assumptions must be made in order to perform a hypothesis test for the difference between these two population means?

b. Will the State Environmental Board conclude at $\alpha = 0.01$ that Service City has a lower pollution level on average than Sunshine City?

c. Repeat part **b.**, assuming that the population variances are not equal.

d. Compare the results of part **b.** and part **c.**

13. In 2009 U.S. charitable giving fell 3.6 percent to $303.75 billion for the year. Total charitable contributions from American individuals, corporations, and foundations fell to $303.75 billion from $315.08 billion for 2008. The largest share of contributions went to religious organizations, representing 33 percent of total giving. The next largest shares went to educational organizations, receiving an estimated 13 percent of the total, and foundations, which received 10 percent of the total. Suppose a sample of 6 employees is randomly chosen from a large corporation and their charitable contributions in 2008 and 2009 are determined. The following table gives these amounts (in dollars). Assume that the population variances are approximately equal.

Charitable Contributions ($)	
Giving in 2008	Giving in 2009
232	215
150	125
50	50
400	350
325	210
175	150

Source: Giving USA Foundation, the Center on Philanthropy at Indiana University.

a. Can we conclude with $\alpha = 0.01$ that the average contribution to charity has decreased in this corporation from 2008 to 2009?

b. Give the assumptions for your test.

c. Repeat part **a.**, assuming that the population variances are not equal.

d. Compare the results of part **a.** and part **c.**

11.3 Paired Difference Test

Suppose we are interested in comparing the durability of the soles of two brands of tennis shoes, Spikes and Kickers. One approach to making this comparison is the independent experimental design discussed in Section 11.1. Using this design one may randomly select 10 people to wear the Spikes brand of shoes for six months and then randomly select 10 other people to wear the Kickers brand of shoes for six months. After the six-month period,