P-values for χ^2 Test Statistics

Similar to the *t*-Test in Section 10.2, we have numerous chi-square distributions, one for each degree of freedom. The chi-square table is constructed such that it provides us with chi-square values for frequently used tail probabilities. The chi-square table in Appendix A, Table G, has only ten tail areas. Thus, in most instances, we will not be able to determine the exact *P*-value. Instead, we will find the closest chi-square values with the appropriate degrees of freedom surrounding the test statistic. For example, in Example 10.6.1, the value of the test statistic was

$$\chi^2 = 56.84$$

which has 29 degrees of freedom. Since the *P*-value is a function of the alternative hypothesis, we write

$$P\text{-value} = P(\chi^2 > 56.84)$$

As stated earlier, we are unable to find the exact probability for the *P*-value so we will need to put bounds on the *P*-value. The excerpt from the chi-square table above Figure 10.6.3 highlights the chi-square values with 29 degrees of freedom. Note that the chi-square table provides the value of the chi-square with the area of α to the right. Also, note that the values of α across the top row decrease from left to right and the chi-square values in the body of the table increase from left to right. At 29 degrees of freedom, we see that the value of the test statistic falls to the right (i.e., it is greater than) of 52.336 at 29 degrees of freedom corresponding to $\alpha = 0.005$. In this case, we report that the *P*-value is less than 0.005. In Example 10.6.1, we conducted the test using $\alpha = 0.01$. Therefore, since the *P*-value is less than 0.005, it is obviously less than 0.01 which would lead us to reject the null hypothesis and conclude that the variance is significantly more than 0.01 mg.

The exact *P*-value can be found using technology. In Example 10.6.1, using technology we obtain a *P*-value of 0.0015, which is less than $\alpha = 0.01$. Again, this leads us to reject the null hypothesis.

10.6 Exercises

Basic Concepts

1. How does testing a hypothesis about a variance differ from testing a hypothesis about a mean?

2. What is the symbol for a critical value for the chi-square distribution? Describe the meaning of this critical value.

Exercises

3. Determine the critical value(s) of the test statistic for each of the following tests for a population variance where the assumption of normality is satisfied.

- **a.** Right-tailed test, $\alpha = 0.01$, $n = 20$
- **b.** Right-tailed test, $\alpha = 0.05$, $n = 24$
- **c.** Right-tailed test, $\alpha = 0.005$, $n = 5$

4. Determine the critical value(s) of the test statistic for each of the following tests for a population variance where the assumption of normality is satisfied.

- **a.** Right-tailed test, $\alpha = 0.025$, $n = 18$
- **b.** Right-tailed test, $\alpha = 0.10$, $n = 24$
- **c.** Right-tailed test, $\alpha = 0.05$, $n = 41$

5. A bolt manufacturer is very concerned about the consistency with which his machines produce bolts that are $\frac{3}{4}$ inch in diameter. When the manufacturing process is working normally the standard deviation of the bolt diameter is 0.05 inch. A random sample of 30 bolts has an average diameter of 0.25 inch with a standard deviation of 0.07 inch.

- **a.** Can the manufacturer conclude that the standard deviation of bolt diameters is greater than 0.05 inches at $\alpha = 0.05$?
- **b.** What assumption did you make about the diameter of the bolts in performing the test in part **a.**?

6. A drug that is used for treating cancer has potentially dangerous side effects if it is taken in doses that are larger than the required dosage for the treatment. The pharmaceutical company that manufactures the drug must be certain that the standard deviation of the drug content in the tablet is not more than 0.1 mg. Twenty-five tablets are randomly selected and the amount of drug in each tablet is measured. The sample has a mean of 20 mg and a variance of 0.015 mg.

- **a.** Do the data suggest at $\alpha = 0.01$ that the standard deviation of drug content in the tablets is greater than 0.1 mg?
- **b.** What assumption did you make about the amount of drug contained in the tablets in performing the test in part **a.**?

7. A conservative investor would like to invest some money in a bond fund. The investor is concerned about the safety of her principal (the original money invested). Colonial Funds claims to have a bond fund which has maintained a consistent share price of $7. They claim that this share price has not varied by more than $0.25 on average since its inception. To test this claim, the investor randomly selects 25 days during the last year and determines the share price for the bond fund. The average share price of the sample is $7 with a standard deviation of $0.35.

- **a.** Can the investor conclude that the standard deviation of share price of the bond fund is greater than 0.25? Test at the 0.01 level.
- **b.** What assumption did you make about the share price of the bond fund in your test in part **a.**?