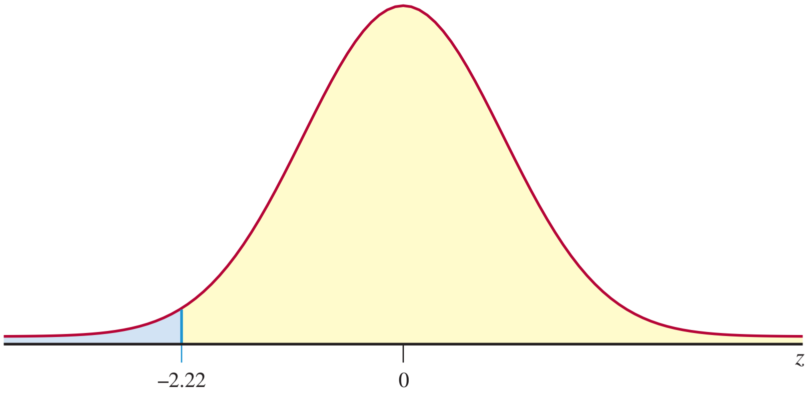
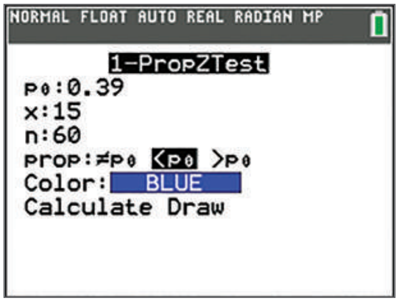
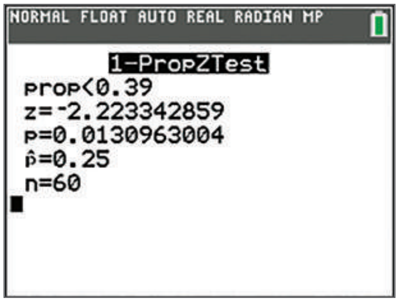


Figure 10.5.3

Technology

P-values for a *z*-test can be found using the standard normal tables or from the output from a hypothesis test using the *z*-test statistic. For instructions on how to conduct a hypothesis test for a proportion, please visit stat.hawkeslearning.com and navigate to **Discovering Business Statistics, Second Edition > Technology Instructions > Hypothesis Testing > One Proportion *z*-Test.**





Assuming the null is true, $\hat{p}$ has a normal distribution that is centered around 0.39. If the null is really true, a *z*-value of −2.22 is uncommon. The probability of observing a value as small or smaller than −2.22 is the *P*-value. Using Table A in the Appendix,

$$P\text{-value} = P(z \le -2.22) = 0.0132.$$

Table 10.5.1 – If *P*-Value = 0.0132

Level of the Test	Reject or Fail to Reject H_0
0.10	Reject
0.05	Reject
0.01	Fail to Reject
0.005	Fail to Reject

If the null is true, the *P*-value measures the rareness of the test statistic under ordinary sampling variation. In other words, how often would we see a test statistic as small or smaller than the test statistic we have observed. Presuming the null is true, ordinary sampling variation produces a test statistic less than or equal to −2.22 about 1 time out of every 100. Should H_0 be rejected? Have we observed a test statistic that is too "rare" for H_0 to be true? The level of the test defines an unacceptable level of rareness for the test statistic. In the previous example $\alpha = 0.01$. Setting $\alpha = 0.01$ implies we are only willing to make a Type I error (reject the null hypothesis when it is true) once in every 100 trials of the experiment. Since the *P*-value of our test statistic, 0.0132, is greater than 0.01, the null hypothesis was not rejected.

If the level of the test had been 0.05, then to reject H_0 in favor of H_a requires a test statistic whose rareness under ordinary sampling variation is less than or equal to 0.05. For $\alpha = 0.05$, the null hypothesis is rejected in favor of the alternative, since the test statistic has a *P*-value (0.0132) less than α. In general, if the *P*-value is less than or equal to the level of the test, α, then H_0 is rejected in favor of H_a. If the level of the test is less than the *P*-value, then H_0 is not rejected.

Note that if H_a had required a two-tailed test we would double the single tail area. Thus, for a test statistic of −2.22 and a two-tailed H_a the resulting *P*-value would be $2(0.0132) = 0.0264$.

10.5 Exercises

Basic Concepts

1. How does testing a hypothesis about a proportion differ from testing a hypothesis about a mean?

2. What is the appropriate test statistic to be used in hypothesis testing of a population proportion?

3. What conditions must be met in order to perform a hypothesis test about a population proportion?

4. How are P-values determined for a proportion?

Exercises

5. Determine the critical value(s) of the test statistic for each of the following large sample tests for the population proportion.

- **a.** Left-tailed test, $\alpha = 0.05$
- **b.** Right-tailed test, $\alpha = 0.01$
- **c.** Two-tailed test, $\alpha = 0.10$

6. Determine the critical value(s) of the test statistic for each of the following large sample tests for the population proportion.

- **a.** Left-tailed test, $\alpha = 0.07$
- **b.** Right-tailed test, $\alpha = 0.04$
- **c.** Two-tailed test, $\alpha = 0.09$

7. A commercial airline is concerned about the increase in usage of carry-on luggage. For years, the percentage of passengers with one or more pieces of carry-on luggage has been stable at approximately 38%. The airline recently selected 300 passengers at random and determined that 148 possessed carry-on luggage. Is there overwhelming evidence of an increase in carry-on luggage at a significance level of 0.01?

8. Ordinarily, when a company recruits a technical staff member, about 25% of the applicants are qualified. However, based on the information in 120 recently received resumes, 18 appear to be technically qualified.

- **a.** Is there overwhelming evidence that the percentage of qualified applicants is less than 25%? Test at the 0.05 level.
- **b.** What concerns might you have about the data in this problem?

9. The National Center for Drug Abuse is conducting a study to determine if heroin usage among teenagers has changed. Historically, about 1.3 percent of teenagers between the ages of 15 and 19 have used heroin one or more times. In a recent survey of 1824 teenagers, 37 indicated they had used heroin one or more times.

- **a.** Is there overwhelming evidence of a change in heroin usage among teenagers? Test at the 0.05 level.
- **b.** What concerns might you have about the data in this problem?

10. Paper International, Inc. has a large staff of salespeople nationwide. Top officials of the company believe that 75% of their salespeople have met their monthly sales goals by the end of the third week of each month. To investigate this, they randomly select 250 salespeople and examine their sales records at the end of the third week of the current month. One-hundred seventy-five of the 250 salespeople surveyed had already met their monthly sales goals.

- **a.** Does this sample support the belief of the top officials at the company at $\alpha = 0.10$?
- **b.** What concerns might you have about the manner in which the data were collected?

11. Ships arriving in U.S. ports are inspected by customs officials for contaminated cargo. Assume, for a certain port, that 20% of the ships arriving in the previous year contained cargo that was contaminated. A random selection of 50 ships in the current year included five that had contaminated cargo.

a. Do the data suggest that the proportion of ships arriving in the port with contaminated cargoes has decreased in the current year at $\alpha = 0.01$?

b. Do you have any concerns about the sample size? Explain.

12. Grain elevators store hundreds of thousands of bushels of grain each year that are waiting to be processed. It is critical to control the amount of moisture in the grain so that it does not spoil. A large storage facility is deemed to be "in control" if 1% of the grain elevators have a moisture content of 10%. One-hundred fifty grain elevators are randomly selected and the moisture content is measured. Two of the grain elevators sampled have a moisture content in excess of 10%.

a. Is there sufficient evidence for the manager to conclude that the storage facility has a moisture content significantly greater than 1%? Use $\alpha = 0.05$.

b. Do you have any concerns about the sample size? Explain.

13. Electronic circuit boards are randomly selected each day to determine if any of the boards are defective. A random sample of 100 boards from one day's production has four boards that are defective.

a. Based on the data, is there overwhelming evidence that more than 5% of the circuit boards are defective? Test at the $\alpha = 0.10$ level.

b. Do you have any concerns about the sample size? Explain.

14. Loch Ness Fish Farm breeds fish for commercial sale. The fish are kept in breeder tanks until more than 70% of the fish are five inches long at which time they are transferred to outdoor ponds. To determine if it is the appropriate time to transfer the fish, 50 fish are randomly selected and measured. If 33 of the fish are found to be over five inches long, does the sample data suggest that it is the appropriate time to transfer the fish at $\alpha = 0.05$?

15. Digger and Digger, a precious metals mining company, is considering the development of a new mining area. They have a lease on an area which they believe contains gypsum. The area will be profitable to mine if more than 15% of the rocks contain more than trace amounts of the mineral. Eighty rocks are randomly selected and the amount of gypsum is measured. Thirteen rocks in the sample are observed to have more than trace amounts of the mineral. Based on the sample data, should Digger and Digger conclude that the area will be profitable to mine? Use $\alpha = 0.01$.

16. A socially conscious corporation wants to relocate their headquarters to another part of town. One concern expressed by workers is that their commuting distance will increase. The corporation has decided that if more than 50% of the employees will have to drive farther to the proposed new location, they will cancel the move. In a random sample of 398 employees, 201 indicated that their commuting distance to the new office will be longer. Based on the sample data, should the corporation cancel the move? Use a significance level of 0.01.

17. A production process will normally produce defective parts 0.2% of the time. In a random sample of 1400 parts, three defectives are observed.

a. Is this overwhelming evidence at the 0.05 level to indicate that the defective rate of the process has increased?

b. Compute the *P*-value for the test statistic.

c. Based on the *P*-value, would the decision change at $\alpha = 0.01$?

18. Bombay Charlie's, a fast food Indian restaurant, is thinking about adding a certain spice to their chicken curry dish to attract more customers. The restaurant manager has decided to add the spice if more than 80% of his customers prefer the taste of the chicken curry with the spice added. Sixty-five customers are randomly selected to participate in a blind taste test. Fifty-four of these customers prefer the chicken curry with the added spice.

- **a.** Find the *P*-value for the hypothesis test that the manager will perform to decide if more than 80% of the customers prefer the taste of the chicken curry with the added spice.
- **b.** Do the data suggest that more than 80% of the customers prefer the curry with the new spice at $\alpha = 0.05$?

19. The news program for KOPE, the local television station, claims to have 40% of the market. A random sample of 500 viewers conducted by an independent testing agency found 192 who claim to watch the KOPE news program on a regular basis.

- **a.** Find the *P*-value for testing the hypothesis that the news program for KOPE does not have more than 40% of the market as it claims.
- **b.** Is there sufficient evidence to reject the hypothesis that KOPE does not have at least 40% of the market at a significance level of 0.05?

20. The length of time that a storm window will last before beginning to leak is of interest to a window manufacturer who wishes to guarantee his windows. He believes that more than 50% of the windows will last at least four years. To research this, 931 windows, which were installed at least four years ago, are randomly selected and checked for leakage. Five hundred of the windows are found to still be leak-free.

- **a.** Find the *P*-value for testing the hypothesis that more than 50% of the windows will be leak-free in four years.
- **b.** Does the sample support the hypothesis that more than 50% of the windows will be leak-free in four years at $\alpha = 0.05$?

21. In order to discourage soldiers from smoking, the Pentagon raised the price of cigarettes by \$4 a carton in October of 1996. This increased the average price of a carton of brand-name cigarettes to \$17.50, an increase of about 30%. Prior to the price increase, about 32% of military personnel smoked, as opposed to 25% of all adult Americans. Suppose that following the price increase, a random sample of military personnel is selected to determine smoking habits. With $\alpha = 0.05$, can we conclude that the price increase was effective in decreasing the percentage of smokers if 50 of the 200 military personnel sampled smoke?

22. Wearing bright or fluorescent orange colored clothing clearly reduces the risk of being shot or killed by hunting. According to an October 1996 article appearing in *The Augusta Chronicle* (Georgia), about two-thirds of the hunters shot in Georgia and South Carolina during the preceding five years were not wearing bright clothes. Of the 52 that were killed, only 19 wore orange. Suppose that a random sample of 100 hunters in Georgia are surveyed and it is determined that of the 100, 62 routinely wear fluorescent orange colored clothing while hunting. With $\alpha = 0.10$, can it be concluded that over half the hunters in Georgia routinely wear fluorescent orange colored clothing while hunting?

23. According to the Federal Communications Commission, about 49% of the households in the United States had cable television in 1985. Suppose that a sample of 200 households is selected in 2003 and it is determined that 125 of them have cable television.

   a. With $\alpha = 0.05$, can it be concluded that a higher proportion of households in 2003 have cable television as compared with 1985?

   b. In the sample of 200, what is the fewest number of people who have cable television that would allow the conclusion that a higher proportion of households in 2003 have cable television as compared to 1985?

24. Selling autographed sports memorabilia has become a multimillion dollar industry in the United States. But just how does the purchaser of an autographed football jersey or an autographed baseball know that the autograph is indeed authentic? Unfortunately, the sports memorabilia market is teeming with con artists who prey upon the trusting nature of sports fans. In 1996, the FBI said that 70% of all autographed sports memorabilia is fraudulent. Assume that 50 pieces of autographed sports memorabilia are sampled at a large memorabilia show and that 40 of them are determined to be fraudulent.

   a. With $\alpha = 0.05$, can it be concluded that the proportion of fraudulent autographed sports memorabilia at the show differs from the FBI claim?

   b. What is the greatest number of fraudulent items in the sample of 50 that would not allow a conclusion that sports memorabilia at the show differs from the FBI claim?

Friedrich Robert Helmert

Friedrich Robert Helmert was born in Germany in 1843. His interests were in geodesy, which is a discipline concerned with measuring the earth on a global scale. He studied engineering science at the Polytechnische Schule and while still a student had the opportunity to work on some important geodesy projects with one of his teachers, August Nagel. He later studied mathematics and astronomy to earn his doctorate. Geodesy led him into statistics, first writing a book on least squares. In 1876 he discovered the chi-square as the distribution of the sample variance for a normal distribution. His work was in German and was not translated to English, so later in 1900 English statisticians rediscovered the chi-square distribution (Karl Pearson) and its application to the sample variance (William Gosset, Ronald Fisher).

10.6 Testing a Hypothesis about a Population Variance

In this section we want to adapt the hypothesis testing procedure to test a hypothesis concerning a population variance. Before we can perform the test about the population variance, we need to review a few topics that were discussed earlier in the text.

Recall that the sample variance is

$$s^2 = \frac{\sum (x_i - \bar{x})^2}{n-1}$$

and it serves as the point estimate of the population variance, σ^2. In Section 9.4 we determined that the sampling distribution of

$$\frac{(n-1)s^2}{\sigma^2}$$

is a chi-square distribution with $n - 1$ degrees of freedom.

Formula

χ^2-Test Statistic

If we have a random sample of size n taken from a normal population, then the sampling distribution of the test statistic is given by

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

which has a chi-square distribution with $n - 1$ degrees of freedom.

To adapt the hypothesis testing procedure to test a hypothesis concerning a population variance, let's look at the following example.