$$H_0: \mu = \$74{,}231$$
$$H_a: \mu \neq \$74{,}231.$$

At the 1% level, a 99% confidence interval for the mean New Yorker salary is obtained by

$$\bar{x} \pm t_{df,\alpha/2} \frac{s}{\sqrt{n}}.$$

We know that the sample size is 3,351, the sample mean, $\bar{x} = \$73{,}707$, and the sample standard deviation is \$2,583. We also know that $\alpha = 0.01$. Thus, $t_{3350,0.005} = 2.577$. Using these data, we find that the 99% confidence interval is given by $73{,}707 \pm (2.577)(2{,}583) / \sqrt{3{,}351}$. Performing the calculations, you will get a confidence interval of (\$73,592, \$73,822). Note that the hypothesized mean of \$74,231 does not fall in the confidence interval above. Thus, we reject the null hypothesis and conclude that the average salary of residents of New York is significantly different than the average salary of residents of Virginia.

10.4 Exercises

Basic Concepts

1. How can a confidence interval be used to test a hypothesis?

Exercises

2. AAA Controls makes a switch that is advertised to activate a warning light if the power supplied to a machine reaches 100 volts. A random sample of 250 switches is tested and the mean voltage at which the warning light occurs is 98 volts with a sample standard deviation of 3 volts. Using the confidence interval approach, test the hypothesis that the mean voltage activation is different from AAA Controls' claim at the 0.05 level.

3. Researchers studying the effects of diet on growth would like to know if a vegetarian diet affects the height of a child. The researchers randomly selected 12 vegetarian children that were six years old. The average height of the children is 42.5 inches with a standard deviation of 3.8 inches. The average height for all six-year-old children is 45.75 inches.

    a. Using confidence intervals, test to determine whether there is overwhelming evidence at $\alpha = 0.05$ that six-year-old vegetarian children are not the same height as other six-year-old children.

    b. What assumption did you make in performing the test?

4. High-power experimental engines are being developed by the Stevens Motor Company for use in its new sports coupe. The engineers have calculated the maximum horsepower for the engine to be 600 HP. Sixteen engines are randomly selected for horsepower testing. The sample has an average maximum HP of 620 with a standard deviation of 50 HP.

    a. Use the confidence interval approach to determine whether the data suggest that the average maximum HP for the experimental engine is significantly different than the maximum horsepower calculated by the engineers. Use a significance level of $\alpha = 0.01$.

    b. What assumption did you make in performing the test?

5. The nutrition label for Oriental Spice Sauce states that one package of sauce has 1190 milligrams of sodium. To determine if the label is accurate, the FDA randomly selects two hundred packages of Oriental Spice Sauce and determines the sodium content. The sample has an average of 1167.34 milligrams of sodium per package with a sample standard deviation of 252.94 milligrams.

   a. Calculate a 99% confidence interval for the mean sodium content in Oriental Spice Sauce.

   b. Using the confidence interval approach, is there evidence that the sodium content is different than the nutrition label states?

6. Officials in charge of televising an international chess competition in South America want to determine if the average time per move for the top players has remained at five minutes over the last two years. Video tapes of matches which have been played over the two-year period are reviewed and a random sample of 50 moves are timed. The sample mean is 3.5 minutes. Assume the population standard deviation is 1.5 minutes. Using the confidence interval approach, test the hypothesis that the average time per move is different from 5 minutes at a 0.01 significance level.

7. In example 9.2.2 of Chapter 9, we found the 95% confidence interval for mean μ, the population average completion time for the stage in the production process, in minutes, to be (20.36, 26.54).

$$n = 10,\ \bar{x} = 23.45,\ s = 4.32$$

   Conduct a hypothesis test, using a 5% level of significance, to see if the population average completion time for the stage in the production process, in minutes, differs from the following values.

   a. The null and the alternate hypotheses are H_0: $\mu = 18.27$ versus H_1: $\mu \neq 18.27$.

   b. The null and the alternate hypotheses are H_0: $\mu = 24.96$ versus H_1: $\mu \neq 24.96$.

   c. The null and the alternate hypotheses are H_0: $\mu = 29.53$ versus H_1: $\mu \neq 29.53$.

8. The owner of an upscale restaurant in Atlanta, Georgia wanted to study the dining characteristics of her customers. She found that in a random sample of 290 customers, 60 purchased dessert. Find a 98% confidence interval for the proportion of customers who purchased dessert. Use this confidence interval to test if the proportion of customers who purchase dessert differs from 25%. Use a 2% level of significance.

9. The chief purchaser for the State Education Commission is reviewing test data for a metal link chain which will be used on children's swing sets in elementary school playgrounds. The average breaking strength for a sample of 50 pieces of chain is 5000 pounds. Based on past experience, the breaking strength of metal chains is known to be normally distributed with a standard deviation of 100 pounds. Estimate the actual mean breaking strength of the metal link chain with 99% confidence. Use this confidence interval to test if the mean breaking strength of the metal link chain is different from 5020 pounds. Use a 1% level of significance.

10. An FDA representative randomly selects 8 packages of ground chuck from a grocery store and measures the fat content (as a percent) of each package. The rating measurements are given below.

Fat Contents							
13%	12%	14%	17%	15%	16%	18%	15%

a. Assuming that the population distribution of the fat content is approximately normal, construct a 90% confidence interval for the true mean fat content of all the packages of ground beef.

b. Use the confidence interval in part (a) to test if the true mean fat content of all the packages of ground beef differs from 17.24%. Use a 10% level of significance.

11. A hospital would like to determine the mean length of stay for its patients having abdominal surgery. A sample of 15 patients revealed a sample mean of 6.4 days and a sample standard deviation of 1.4 days. Assume that the lengths of stay are approximately normally distributed.

a. Construct a 95% confidence interval for the mean length of stay for patients with abdominal surgery.

b. Use the confidence interval in part (a) to test if the mean length of stay for patients having abdominal surgery differs from 5.4 days. Use a 5% level of significance.

10.5 Testing a Hypothesis about a Population Proportion

The topic that we will develop in this section will be a hypothesis testing approach for categorical values (nominal data). The inferences that we will make with these data will concern one population. We will use the information in the sample proportion ($\hat{p}$) to test hypotheses about the population proportion, p.

Testing hypotheses about a population proportion could involve a variety of problems.

- What fraction of a student's grades will be A's?
- What fraction of graduating seniors obtain jobs with starting salaries in excess of $38,000?
- What fraction of products that a company produces are defective?
- What fraction of the voters favor the incumbent in the next election?
- What fraction of the customers who purchase a Ford Focus are extremely satisfied?
- What fraction of the time will a baseball player get a hit?
- What fraction of the time will a drug be successful in treating a specific disorder?

Developing the Test

Testing a hypothesis concerning a population proportion is nearly identical to testing a hypothesis about a population mean. The major changes in the procedure include the use of the population proportion (p) in the formulation of the hypotheses rather than the population mean (μ), and the calculation of the test statistic. Let's try an example.