SOLUTION

To test Benny's claim, the hypothesis test is carried out with the following hypotheses.

$$H_0: \mu = 25 \text{ mph}$$
$$H_a: \mu > 25 \text{ mph}$$

The resulting test statistic is

$$z = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{25.01 - 25}{\frac{0.10}{\sqrt{2000}}} = 4.47.$$

The P-value for this test is 0.000004 and suggests that the test statistic is extremely rare, if the null hypothesis is true. Considering that a test statistic this large would result from ordinary sampling variation in only about 4 in a million samples, we should reject the null hypothesis (H_0: $\mu = 25$ mph) in favor of the alternative. With Benny being a "strictly by the book" officer, he would conclude that the speeds are significantly higher than 25 mph in the school zone and the department should allocate extra resources in that area to reduce speeds. However, from a practical perspective, there isn't much difference (other than random variation) between 25 mph and 25.01 mph, and it isn't likely to be detected by radar. So, despite the "statistical significance" of the test, the practical significance is negligible.

It is important to keep the practical significance of the hypothesis test in mind when making conclusions. This is one of the reasons why in the 6-step procedure for hypothesis testing, **Step 6** is to state the conclusion in terms of the original problem. This step may help shed light on the practical significance of the hypothesis test.

10.3 Exercises

Basic Concepts

1. Suppose a null hypothesis were rejected at $\alpha = 0.05$. Would it be rejected at 0.10? Explain.
2. Suppose a null hypothesis were rejected at $\alpha = 0.05$. Would it be rejected at 0.01? Explain.
3. What is a P-value?
4. Discuss how P-values are used in the test of a hypothesis.
5. Describe the difference between statistical significance and practical significance.
6. Give an example of a situation in which results could be statistically significant but not practically significant.

Exercises

7. Determine the critical value(s) of the test statistic for each of the following tests for the population mean where the population standard deviation is unknown and the assumption of normality is satisfied.
    - **a.** Left-tailed test, $\alpha = 0.01$, $n = 15$
    - **b.** Right-tailed test, $\alpha = 0.10$, $n = 20$
    - **c.** Two-tailed test, $\alpha = 0.05$, $n = 8$

8. Determine the critical value(s) of the test statistic for each of the following tests for the population mean where the population standard deviation is unknown and the assumption of normality is satisfied.

a. Left-tailed test, $\alpha = 0.005$, $n = 12$

b. Right-tailed test, $\alpha = 0.025$, $n = 5$

c. Two-tailed test, $\alpha = 0.10$, $n = 25$

9. Consider the following random sample of size six from a normal population. Based on the sample, perform a hypothesis test to test the claim that the mean of the population is not equal to 10 at $\alpha = 0.05$.

10	15	12	9	11	10

10. Consider the following random sample of size eight from a normal population. Based on the sample, perform a hypothesis test to test the claim that the mean of the population is greater than 100 at $\alpha = 0.05$. Calculate the P-value for this hypothesis test.

100	150	120	90	95	110	100	80

11. Consider the following random sample of size seven from a normal population. Based on the sample, perform a hypothesis test to test the claim that the mean of the population is less than 0.5 at $\alpha = 0.10$. Calculate the P-value for this hypothesis test.

0.3	0.5	0.4	0.6	0.5	0.4	0.4

12. NarStor, a computer disk drive manufacturer, claims that the average time to failure for its hard drives is 14,400 hours. You work for a consumer group that has decided to examine this claim. Technicians ran 16 drives continuously for three years. Recently the last drive failed. The time to failure (in hours) are given below.

Time Until Failure (Hours)							
330	620	1870	2410	4620	6396	7822	8102
8309	12,882	14,419	16,092	18,384	20,916	23,812	25,814

a. What is the population being studied?

b. What is the variable being measured?

c. What level of measurement does the variable possess?

d. Conduct a hypothesis test to determine whether there is overwhelming evidence that the average time to failure is less than the manufacturer's claim. Use $\alpha = 0.01$.

e. What assumption did you make in performing the test in part **d.**?

13. The admitting office at Sisters of Mercy Hospital wants to be able to inform patients of the average level of expenses they can expect per day. Historically, the average has been approximately \$1240. The office would like to know if there is evidence of an increase in the average daily billing. Twenty randomly selected patients have an average daily charge of \$1491 with a standard deviation of \$342.

a. What is the population being studied?

b. Conduct a hypothesis test to determine whether there is evidence that average daily charges have increased at $\alpha = 0.10$.

c. What assumption did you make in performing the test in part **b.**?

14. A supplier has agreed to provide the manager of a large hospital with light bulbs that he claims will last more than 1000 hours. Twenty-five bulbs are randomly selected and tested by the hospital's maintenance department. The sample has an average life of 1099 hours with a standard deviation of 99 hours.

- **a.** Perform a hypothesis test to determine whether the data support the supplier's claim at $\alpha = 0.05$.
- **b.** What assumption did you make in performing the test in part **a.**?
- **c.** What is the *P*-value for the hypothesis test performed in part **a.**?

15. The managers of a large department store wish to test reactions of shoppers to a new in-store video screen which will broadcast continuous information about the store and the items currently on sale. In past promotions, the video production company has indicated that the average shopper watched for five minutes. The managers randomly select 17 shoppers and determine how long they watch the video. The average time is 4.5 minutes with a standard deviation of 2.5 minutes.

- **a.** Perform a hypothesis test to determine whether there is overwhelming evidence to indicate that the shoppers will watch for less than five minutes. Use $\alpha = 0.01$.
- **b.** What assumption did you make in performing the test in part **a.**?

16. A group of local businessmen is thinking about developing land into a shopping mall. To evaluate the desirability of the location, they count the number of shoppers who visit the neighboring shopping center each day. A random sample of 25 days reveals a daily average of 107 shoppers with a standard deviation of 23 shoppers. The businessmen will develop the land if the average number of shoppers per day is more than 100.

- **a.** Based on the sample data, should the businessmen develop the land? Perform a hypothesis test and use $\alpha = 0.10$.
- **b.** What assumption did you make in performing the test in part **a.**?

17. The Dodge Reports are used by many companies in the construction field to estimate the time required to complete various jobs. The company has received several complaints that the time required to install 130 square feet of bathroom tile is greater than the eight hours reported in the current manual. A researcher for Dodge randomly selects 10 construction workers and determines the time required to install 130 square feet of bath tile. The average time required to install the tile for the sample is 8.5 hours with a standard deviation of 1 hour.

- **a.** Use a hypothesis test to determine whether the customers' complaints are substantiated by the data. Use $\alpha = 0.05$.
- **b.** What assumption did you make in performing the test in part **a.**?

18. Officials in charge of televising an international chess competition in South America want to determine if the average time per move for the top players has remained under five minutes over the last two years. Video tapes of matches which have been played over the two-year period are reviewed and a random sample of 50 moves are timed. The sample mean is 3.5 minutes with a standard deviation of 1.5 minutes.

- **a.** What is the population under study?
- **b.** Can the officials conclude at $\alpha = 0.05$ that the time per move is still under five minutes?

19. Buckshot Heaven is developing a new shotgun shell that they hope will have a significantly tighter pellet pattern than their competition. Twenty-five shells are tested at fifty yards. The average pellet pattern of the sample was 8.7 inches in diameter with a standard deviation of 2.0 inches. Their competitor advertises that the average pellet pattern of their shells is nine inches.

- **a.** Does the test completed by Buckshot Heaven support the claim that their shell pattern is tighter than the competition at a level of significance of 0.10?
- **b.** What assumption did you make in performing the test in part **a.**?

20. For each of the following combinations of the *P*-value and α, decide whether you would reject or fail to reject the null hypothesis.

- **a.** *P*-value = 0.0839, $\alpha = 0.05$
- **b.** *P*-value = 0.0174, $\alpha = 0.02$
- **c.** *P*-value = 0.0444, $\alpha = 0.10$
- **d.** *P*-value = 0.0374, $\alpha = 0.01$

21. Consider the following hypothesis tests for the population mean. Compute the *P*-value for each test and decide whether you would reject or fail to reject the null hypothesis at $\alpha = 0.01$. See the Discovering Technology section at the end of this chapter for instructions on finding exact *P*-values for *t*-statistics.

- **a.** H_0: $\mu = 25$, H_a: $\mu > 25$, $t = 2.7$, $n = 15$
- **b.** H_0: $\mu = 0.85$, H_a: $\mu < 0.85$, $t = -2.5$, $n = 7$
- **c.** H_0: $\mu = 1000$, H_a: $\mu \neq 1000$, $t = 2.0$, $n = 15$

22. Consider the following small sample hypothesis tests for the population mean. Compute the *P*-value for each test and decide whether you would reject or fail to reject the null hypothesis at $\alpha = 0.05$. See the Discovering Technology section at the end of this chapter for instructions on finding exact *P*-values for *t*-statistics.

- **a.** H_0: $\mu = 120$, H_a: $\mu > 120$, $t = 1.5$, $n = 20$
- **b.** H_0: $\mu = 0.2$, H_a: $\mu < 0.2$, $t = -2.75$, $n = 18$
- **c.** H_0: $\mu = 50$, H_a: $\mu \neq 50$, $t = 2.4$, $n = 5$

23. A.C. Bone has developed a duck hunting boot which it claims can remain immersed for more than 12 hours without leaking. Five hundred pairs of the boots are tested and the time until first leakage is measured. The average time until first leakage for the sample is 12.25 hours with a standard deviation of 3.0 hours.

- **a.** Find the *P*-value to test the claim that the average time until first leakage for the hunting boot is more than 12 hours.
- **b.** Does this sample support A.C. Bone's claim at $\alpha = 0.10$?

24. In preparation for upcoming wage negotiations with the union, the managers for the Bevel Hardware Company want to establish the time required to assemble a kitchen cabinet. A first line supervisor believes that the job should take 45 minutes on average to complete. A random sample of 125 cabinets has an average assembly time of 47 minutes with a standard deviation of 10 minutes. Is there overwhelming evidence to contradict the first line supervisor's belief at a 0.05 significance level?

Discuss the statistical and practical significance for this problem.

25. A horticulturist working for a large plant nursery is conducting experiments on the growth rate of a new shrub. Based on previous research, the horticulturist feels the average daily growth rate of the new shrub is 1 cm per day. A random sample of 45 shrubs has an average growth of 0.90 cm per day with a standard deviation of 0.30 cm. Will a test of hypothesis at the 0.05 significance level support the claim that the growth rate is less than 1 cm per day?

Discuss the statistical and practical significance for this problem.

26. The director of the IRS has been flooded with complaints that people must wait more than 45 minutes before seeing an IRS representative. To determine the validity of these complaints, the IRS randomly selects 400 people entering IRS offices across the country and records the times which they must wait before seeing an IRS representative. The average waiting time for the sample is 55 minutes with a standard deviation of 15 minutes. Are the complaints substantiated by the data at $\alpha = 0.10$?

Discuss the statistical and practical significance for this problem.

27. The managers of a large department store wish to test reactions of shoppers to a new in-store video screen which will broadcast continuous information about the store and the items currently on sale. In past promotions, the video production company has indicated that the average shopper watched for five minutes. The managers randomly select 17 shoppers and determine how long they watch the video. The average time is 4.5 minutes with a standard deviation of 2.5 minutes. Perform a hypothesis test to determine whether there is overwhelming evidence to indicate that the shoppers watch for less than five minutes. Use $\alpha = 0.01$.

Discuss the statistical and practical significance for this problem.

10.4 The Relationship Between Confidence Interval Estimation and Hypothesis Testing

Previously, we discussed interval estimation for the population mean and the population proportion. We know that when estimating the population mean, a $100(1-\alpha)\%$ confidence interval for μ is given by

$$\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}.$$

In this chapter, we have shown that the two-sided hypothesis test about the population mean μ is

$$H_0\colon \mu = \mu_0$$

$$H_a\colon \mu \neq \mu_0$$

where μ_0 is some specific value of the population mean.

From the previous chapter, we know that $100(1-\alpha)\%$ of the confidence intervals will contain μ. Therefore, if we reject the null hypothesis when the confidence interval does not contain the value of μ_0, we will reject the null hypothesis when it is actually true with probability of α. You may recall that α represents the probability of committing a Type I error (i.e., we reject the null hypothesis when the null hypothesis is true). So, when we construct a $100(1-\alpha)\%$ confidence interval and reject the null hypothesis when the interval does not contain μ_0, this is equivalent to performing a two-tailed hypothesis test using α as the level of the test.