

A Procedure for Testing a Hypothesis

A procedure for testing a hypothesis is given below. In examining this procedure, you will notice we have already discussed the first two steps. The next four steps assist us in defining the **decision rule**, which is a criterion used to determine whether the null or the alternative hypothesis will be chosen. As you look over these steps, do not be overly concerned if you do not understand everything. You will learn by following the examples.

Steps in the Test of a Hypothesis

Step 1: Determine the null hypothesis. In this process, select the appropriate statistical measure, such as the population mean, proportion, or variance.

Step 2: Determine the alternative hypothesis and whether it should be one-sided or two-sided.

Step 3: Select the appropriate test statistic based on the information at hand and the assumptions you are willing to make.

Step 4: Determine the critical value of the test statistic. Two factors must be considered.

1. The type of alternative hypothesis: two-sided, one-sided left, one-sided right. If the alternative hypothesis is two-sided, the hypothesis test will be **two-tailed**. If the alternative hypothesis is one-sided left, the hypothesis test will be a **left-tailed** or **lower-tailed** test. If the alternative hypothesis is one-sided right, the hypothesis test will be a **right-tailed** or **upper-tailed** test.
2. The specification of α , the significance level of the test.

Step 5: Collect the sample data and compute the value of the test statistic.

Step 6: Make the decision and state the conclusion in terms of the original question.

- If the value of the test statistic is in the rejection region, reject the null hypothesis in favor of the alternative.
- If the value of the test statistic is not in the rejection region, fail to reject the null hypothesis.

NOTE

Note at **Step 4** there are two options; you can find the critical value of the test statistic or the P -value of the test statistic. Both methods will always produce equivalent results; meaning, the decision regarding the hypothesis test will always be the same with both methods. We will often cover both methods in an example to illustrate this. Even though we may show a critical value and a P -value, only one of these is required to make the decision to reject or fail to reject the null hypothesis. You or your instructor may have a preference of one method over another.

Type I and Type II Errors in the Trial of the Pyx

If the coins did in fact weigh less than they were intended to, the currency would become debased, and the Mint would be making a profit because they would be pocketing some of the metals they should be turning into coins. If the coins weighed more than they were supposed to, someone could collect these overweight coins, and sell them back to the Mint for a profit. Either way, the king is not happy that someone besides him is able to profit. And in those days, if the king is not happy, there is a high likelihood of important body parts being involuntarily cut off. So, if the coins are found to be off from the standard value, it could mean serious consequences for the head of the Mint.

The hypotheses are set up such that a Type I error implies that the coins were believed to be off from the standard value, when in fact they were meeting the standard. A Type II error would mean that the panel has believed the coins to be matching the weight standard, when in fact they are overweight or underweight. This is the preferred formulation for the head of the Mint, as the Type I error is the one he would like to control so that he does not lose his extremities for no reason! A Type II error would serve the Mint well, as it means the coins were in error, but it went undetected.

10.1 Exercises

Basic Concepts

1. What is a hypothesis?
2. What is the first step in the test of a hypothesis?
3. Describe the common elements present in all hypothesis tests.
4. Summarize the difference between the null and alternative hypotheses.
5. Define and give an example of a one-sided alternative. How does this differ from a two-sided alternative?
6. What is the connection between one and two-sided alternatives and one and two-tailed tests?
7. Is there a way to be absolutely certain your decision is correct when performing a hypothesis test? Explain.
8. What are the three important things you must be able to do in order to be successful at formulating hypothesis testing problems?
9. Describe a Type I error.

10. Describe a Type II error.
11. Explain how Type I and Type II errors influence the construction of a hypothesis.
12. Can both Type I and Type II errors be controlled in the hypothesis testing procedure? Explain.
13. What is the level of the test?
14. Why is a Type II error difficult to express numerically?

Exercises

15. The town mayor believes that more than 47% of the town residents favor annexation of a new community. How should she formulate the hypotheses to test her claim?
16. A chocolate chip manufacturer would like to know if its bag filling machine works correctly at the 450 gram setting. Assume the population is normally distributed. How should the manufacturer formulate the hypotheses to test if the bags are being overfilled?
17. A hospital director believes that 29% of the lab reports contain errors and feels an audit is required. A sample of 300 reports found 99 errors. Is there sufficient evidence at the 0.02 level to refute the hospital director's claim? State the null and alternate hypotheses for this test.
18. An engineer has designed a valve that will regulate water pressure on an automobile engine. The valve was tested on 140 engines and the mean pressure was 7.7 lbs/square inch. Assume the variance is known to be 0.64. If the valve was designed to produce a mean pressure of 7.9 lbs/square inch, is there sufficient evidence at the 0.10 level that the valve performs below the specifications? State the null and alternative hypotheses.
19. Using traditional methods it takes 10.9 hours to receive a basic flying license. A new license training method using Computer Aided Instruction (CAI) has been proposed. Set up the hypotheses to test the claim at the 0.05 level that the new technique performs differently than the traditional method. State the null and alternative hypotheses.
20. Our environment is very sensitive to the amount of ozone in the upper atmosphere. The level of ozone normally found is 7.6 parts/million (ppm). A researcher believes that the current ozone level is higher than the normal level. Set up the hypotheses to test the researcher's claim.
21. An automobile manufacturer claims that their van has a 56.8 miles/gallon (MPG) rating. An independent testing firm has been contracted to test the MPG for this van. After testing 99 vans they found a mean MPG of 56.4 with a standard deviation of 1.2 MPG. Is there sufficient evidence at the 0.025 level that the vans underperform the manufacturer's MPG rating? State the null and alternative hypotheses for this test.
22. A restaurant owner believes that tardiness has become a problem with her staff. In past years around 5% of her employees showed up late for their shift. She believes that the current rate is much higher. How should she formulate the hypotheses to test her belief?
23. For the following situations, develop the appropriate H_0 and H_a and state what the consequences would be for Type I and Type II errors.
 - a. The Standard Tire Company has introduced a new tire in Europe that will be guaranteed to last at least 30,000 kilometers. Standard Tire has hired an independent agency to determine if there is overwhelming evidence that their tires will last through the warranty period.
 - b. Mrs. Russell, head product tester for Hathaway Tool Corporation, is testing a newly designed series of bar hooks. The hooks have been designed to give way if they get too hot. The previous design gave way at 240 degrees. Develop a test to determine if the newly designed hooks give way at a higher temperature than the previous design.

24. For the following situations, develop the appropriate H_0 and H_a and state what the consequences would be for Type I and Type II errors.
- A company that manufactures one-half inch bolts selects a random sample of bolts to determine if the diameter of the bolts differs significantly from the required one-half inch.
 - A company that manufactures safety flares randomly selects 100 flares to determine if the flares last at least three hours on average.
 - A consumer group believes that a new sports coupe gets significantly fewer miles to the gallon than advertised on the sales sticker. To confirm this belief, they randomly select several of the new coupes and measure the miles per gallon.

10.2 Testing a Hypothesis about a Population Mean, σ Known

The following example will be used to illustrate the hypothesis testing procedure. The example goes through all the steps and contains detailed explanations of several new concepts.

Suppose that the average amount of money a student spends on textbooks per semester at college campuses in the U.S. is \$500 with a standard deviation of \$100. A local university wants to know if its students are actually spending that amount on textbooks. The level of the test is to be set at 0.05. A random sample of 75 college students has been selected and the resulting average is \$540 with a sample standard deviation of \$95 spent per semester on textbooks.

Example 10.2.1

Performing a Hypothesis Test for the Mean Cost of Textbooks

SOLUTION

Step 1: Determine the null hypothesis. In this process, select the appropriate statistical measure, such as the population mean, proportion, or variance.

The null hypothesis is fairly straightforward. The local university's students are spending an average of \$500 per semester on textbooks. Thus, the statistical measure is the population mean which is

$\mu =$ the average amount spent on textbooks per semester by college students at the local university.

The null hypothesis should be written as $H_0: \mu = \$500$.

Step 2: Determine the alternative hypothesis and whether it should be one-sided or two-sided.

The alternative hypothesis is that the students are not spending an average of \$500 per semester on textbooks. Nothing is mentioned in the problem to indicate that the university is interested in learning if its students are spending more than the national average per semester on textbooks, nor is there anything mentioned that the university is interested in learning if its students are spending less than the national average per semester on textbooks. Thus, it is assumed that the university is interested in determining if there is any departure from the national average. Consequently, the problem needs to be formulated as a two-sided alternative which should be written as $H_a: \mu \neq \$500$.