

**Example 6 Simplifying Powers of  $i$** Simplify each power of  $i$ .

$$\text{a. } i^{45} = i^{44} \cdot i = (i^4)^{11} \cdot i = 1^{11} \cdot i = i \quad i = 0 + i \text{ in standard form.}$$

$$\text{b. } i^{58} = i^{56} \cdot i^2 = (i^4)^{14} \cdot i^2 = 1^{14} \cdot (-1) = -1 \quad 1 = -1 + 0i \text{ in standard form.}$$

$$\text{c. } i^{-7} = \frac{1}{i^7} = \frac{1}{i^7} \cdot \frac{i}{i} = \frac{i}{i^8} = \frac{i}{1} = i \quad i = 0 + i \text{ in standard form.}$$

6. Simplify each power of  $i$ .

a.  $i^{37}$

b.  $i^{14}$

c.  $i^{-8}$

**Now work margin exercise 6.****Margin Exercise Answers**

1. a.  $10i$  b.  $7i$  c.  $3i\sqrt{2}$  d.  $6i\sqrt{2}$  2. a. real: 0; imaginary: 5 b. real: 14; imaginary:  $\sqrt{7}$

c. real:  $\frac{6}{5}$ ; imaginary:  $-\frac{11}{5}$  d. real: -13; imaginary: 0 3. a.  $x = 10$  and  $y = -2$

b.  $y = 1$  and  $x = -5$  4. a.  $7 + 4i$  b.  $-4\sqrt{2}i$  c.  $28 + 4i$  d.  $2 + 26i$  5. a.  $\frac{8}{5} + \frac{16}{5}i$

b.  $\frac{1}{9} - \frac{4\sqrt{5}}{9}i$  c.  $-\frac{1}{4} - i$  6. a.  $i$  b.  $-1$  c.  $1$

## 9.8 Exercises

### Concept Check

**Fill-in-the-Blank.** Complete each sentence using information found in this section.

- The square roots of negative numbers are not real numbers but are \_\_\_\_\_ numbers.
- Complex numbers consist of two parts: a/an \_\_\_\_\_ part and a/an \_\_\_\_\_ part.
- The standard form of a complex number is \_\_\_\_\_.
- Adding and subtracting complex numbers is similar to adding and subtracting \_\_\_\_\_.
- The expressions  $a + bi$  and  $a - bi$  are \_\_\_\_\_ of each other.
- There are 4 possible values for any power of  $i$ : \_\_\_\_, \_\_\_\_, \_\_\_\_, and \_\_\_\_.

**True/False.** Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

- If  $a + bi = c + di$ , then  $a = d$  and  $b = c$ .
- The square root of negative one is one.
- The conjugate of  $4 - 5i$  is  $4 + 5i$ .
- When  $i$  is squared, the product is 1.

## Practice

Simplify each radical. See Example 1.

1.  $\sqrt{-49}$

2.  $\sqrt{-121}$

3.  $-\sqrt{-64}$

4.  $-\sqrt{-169}$

5.  $\sqrt{147}$

6.  $\sqrt{128}$

7.  $2\sqrt{-150}$

8.  $4\sqrt{-99}$

9.  $-2\sqrt{-108}$

10.  $2\sqrt{175}$

Find the real part and the imaginary part of each of the complex numbers. See Example 2.

11.  $4 - 3i$

12.  $\frac{3}{4} + i$

13.  $-11 + \sqrt{2}i$

14.  $6 + \sqrt{3}i$

15.  $\frac{3}{8}$

16.  $\frac{4}{7}i$

17.  $\frac{4 + 7i}{5}$

18.  $\frac{2 - i}{4}$

19.  $\frac{2}{3} + \sqrt{17}i$

20.  $-\sqrt{5} + \frac{\sqrt{2}}{2}i$

Solve the equations for  $x$  and  $y$ . See Example 3.

21.  $x + 3i = 6 - yi$

22.  $2x - 8i = -2 + 4yi$

23.  $\sqrt{2} + i - 3 = x + yi$

24.  $\sqrt{5}i - 3 + 4i = x + yi$

25.  $3x + 2 - 7i = i - 2yi + 5$

26.  $x + yi + 8 = 2i + 4 - 3yi$

27.  $x + 2i = 5 - yi - 3 - 4i$

28.  $2x + 3 + 6i = 7 - yi - 2i$

29.  $2 + 3i + x = 5 - 7i + yi$

30.  $11i - 2x + 4 = 10 - 3i + 2yi$

Perform the indicated operations and write each result in standard form. See Example 4.

31.  $(2 + 3i) + (4 - i)$

32.  $(7 - i) + (3 + 6i)$

33.  $(4 + 5i) - (3 - 2i)$

34.  $(-3 + 2i) - (6 + 2i)$

35.  $(4 - 3i) + (2 - 3i)$

36.  $(7 + 5i) + (6 - 2i)$

37.  $(8 + 9i) - (8 - 5i)$

38.  $(-6 + i) - (2 + 3i)$

39.  $(\sqrt{5} - 2i) + (3 - 4i)$

40.  $(4 + 3i) - (\sqrt{2} + 3i)$

41.  $(7 + \sqrt{6}i) + (-2 + i)$

42.  $(\sqrt{11} + 2i) + (5 - 7i)$       47.  $(13 - 3\sqrt{-16}) + (-2 - 4\sqrt{-1})$   
 43.  $(\sqrt{3} + \sqrt{2}i) - (5 + \sqrt{2}i)$       48.  $(7 + \sqrt{-9}) - (3 - 2\sqrt{-25})$   
 44.  $(\sqrt{5} + \sqrt{3}i) + (1 - i)$       49.  $(4 + i) + (-3 - 2i) - (-1 - i)$   
 45.  $(5 + \sqrt{-25}) - (7 + \sqrt{-100})$       50.  $(-2 - 3i) + (6 + i) - (2 + 5i)$   
 46.  $(1 + \sqrt{-36}) - (-4 - \sqrt{-49})$

Perform the indicated operations and write each result in standard form. See Examples 4 and 5.

51.  $8(2 + 3i)$       62.  $\frac{-3}{2i}$   
 52.  $-3(7 - 4i)$       63.  $\frac{-4}{1 + 2i}$   
 53.  $(5 + 3i)(1 + i)$       64.  $\frac{7}{5 - 2i}$   
 54.  $(2 + 7i)(6 + i)$       65.  $\frac{2 - i}{2 + 5i}$   
 55.  $(5 + 7i)^2$       66.  $\frac{6 + i}{3 - 4i}$   
 56.  $(3 + 2i)^2$       67.  $\frac{2 - 3i}{-1 + 5i}$   
 57.  $(4 + \sqrt{5}i)(4 - \sqrt{5}i)$       68.  $\frac{-3 + i}{7 - 2i}$   
 58.  $(7 + 2\sqrt{3}i)(7 - 2\sqrt{3}i)$       69.  $\frac{\sqrt{3} + 2i}{\sqrt{3} - 2i}$   
 59.  $(3 + \sqrt{5}i)(3 + \sqrt{6}i)$       70.  $\frac{\sqrt{6} - 3i}{\sqrt{6} + 3i}$   
 60.  $(2 - \sqrt{3}i)(3 - \sqrt{2}i)$   
 61.  $\frac{5}{4i}$

Simplify the following powers of  $i$  and write each result in standard form. Assume  $k$  is a positive integer. See Example 6.

71.  $i^{13}$       76.  $i^{-5}$   
 72.  $i^{20}$       77.  $i^{4k}$   
 73.  $i^{30}$       78.  $i^{4k+2}$   
 74.  $i^{15}$       79.  $i^{4k+3}$   
 75.  $i^{-3}$       80.  $i^{4k+1}$

Find the indicated products and simplify.

81.  $(x + 3i)(x - 3i)$       84.  $(2x + \sqrt{7}i)(2x - \sqrt{7}i)$   
 82.  $(y + 5i)(y - 5i)$       85.  $(\sqrt{5}y + 2i)(\sqrt{5}y - 2i)$   
 83.  $(x + \sqrt{2}i)(x - \sqrt{2}i)$       86.  $(y - \sqrt{3}i)(y + \sqrt{3}i)$

87.  $[(x+2)+6i][(x+2)-6i]$

89.  $[(y-3)+2i][(y-3)-2i]$

88.  $[(x+1)-\sqrt{8}i][(x+1)+\sqrt{8}i]$

90.  $[(x-1)+5i][(x-1)-5i]$

## Writing & Thinking

91. Answer the following questions and give a brief explanation of your answer.
- Is every real number a complex number?
  - Is every complex number a real number?
92. Explain why the product of every complex number and its conjugate is a nonnegative real number.
93. Explain why  $\sqrt{-4} \cdot \sqrt{-4} \neq 4$ . What is the correct value of  $\sqrt{-4} \cdot \sqrt{-4}$ ?