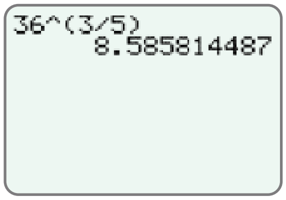
Step 4: Press [ENTER].

The display should appear as follows.



Now work margin exercise 5.

Margin Exercise Answers

1. a. 6 **b.** 2 **c.** -3 **d.** 0.2 **e.** not a real number **2. a.** $\sqrt[5]{x^2}$ **b.** $8\sqrt[7]{z^6}$ **c.** $-\sqrt[4]{b^5}$ **d.** $x^{\frac{5}{7}}$ **e.** $3s^{\frac{1}{2}}$ **f.** $-5^{\frac{1}{4}}$ **3. a.** $x^{\frac{7}{12}}$ **b.** $\frac{1}{a^{\frac{2}{9}}}$ **c.** $81b^{\frac{4}{5}}$ **d.** $\frac{z^{\frac{2}{9}}}{8}$ **e.** not a real number **f.** 4 **4. a.** $\sqrt[10]{x}$ **b.** $\sqrt[4]{x^3}$ **c.** $x\sqrt[3]{x}$ **5. a.** 256 **b.** 22.52722735

9.3 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. The expression $\sqrt[n]{a}$ is called a/an __________ expression, and n is called the __________.

2. In the expression $-\sqrt[5]{3}$, the coefficient is ___ and the index is ___.

3. The expression $\sqrt{a}$ would be rewritten as _____ in exponential notation.

4. An equivalent way to write $a^{\frac{m}{n}}$ is $\left(a^{\frac{1}{n}}\right)^m$ or ______.

5. The expression $a^{\frac{1}{3}}$ can be written as ____ in radical notation.

6. For n^{th} roots, if $b = \sqrt[n]{a}$, then $b =$ ____.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. The same rules for exponents apply to both integer exponents and rational exponents.

8. If the cube root of 7 were to be converted into exponential notation it would be $\sqrt[3]{7}$.

9. Any expression to the power 0, such as $\left(\sqrt[4]{x}\right)^0$, is equal to 1.

10. The expression $y^{\frac{1}{2}}$ can be rewritten in radical notation as $\sqrt{y^2}$.

Practice

Write an equivalent expression using radical notation. See Example 2.

1. $8^{\frac{1}{3}}$
2. $5^{\frac{1}{2}}$
3. $-x^{\frac{1}{6}}$
4. $4y^{\frac{3}{4}}$
5. $(2z)^{\frac{2}{5}}$

Write an equivalent expression using exponential notation. See Example 2.

6. $\sqrt{3}$
7. $\sqrt[7]{13}$
8. $4\sqrt[3]{x^2}$
9. $\sqrt[3]{-9}$
10. $\sqrt[5]{16x^2}$

Simplify each numerical expression. See Example 3.

11. $9^{\frac{1}{2}}$
12. $121^{\frac{1}{2}}$
13. $100^{-\frac{1}{2}}$
14. $25^{-\frac{1}{2}}$
15. $-64^{\frac{3}{2}}$
16. $(-64)^{\frac{3}{2}}$
17. $(-64)^{\frac{1}{3}}$
18. $-(64)^{\frac{1}{3}}$
19. $\left(-\frac{4}{25}\right)^{\frac{1}{2}}$
20. $-\left(\frac{4}{25}\right)^{\frac{1}{2}}$
21. $\left(\frac{9}{49}\right)^{\frac{1}{2}}$
22. $\left(\frac{225}{144}\right)^{\frac{1}{2}}$
23. $64^{\frac{2}{3}}$
24. $8^{-\frac{2}{3}}$
25. $(-216)^{-\frac{1}{3}}$
26. $(-125)^{\frac{1}{3}}$
27. $\left(\frac{8}{125}\right)^{-\frac{1}{3}}$
28. $-\left(\frac{16}{81}\right)^{-\frac{3}{4}}$
29. $\left(-\frac{1}{32}\right)^{\frac{2}{5}}$
30. $\left(\frac{27}{64}\right)^{\frac{2}{3}}$
31. $3 \cdot 16^{-\frac{3}{4}}$
32. $2 \cdot 25^{-\frac{1}{2}}$
33. $-100^{-\frac{3}{2}}$
34. $-49^{-\frac{5}{2}}$
35. $\left[\left(\frac{1}{32}\right)^{\frac{2}{5}}\right]^{-3}$
36. $\left[(-27)^{\frac{2}{3}}\right]^{-2}$

Use a graphing calculator to find the value of each numerical expression accurate to the nearest ten-thousandth, if necessary. See Example 5.

37. $25^{\frac{2}{3}}$
38. $81^{\frac{7}{4}}$
39. $100^{\frac{7}{2}}$
40. $100^{\frac{1}{3}}$
41. $250^{\frac{5}{6}}$
42. $2000^{\frac{2}{3}}$
43. $24^{-\frac{3}{4}}$
44. $18^{-\frac{3}{2}}$
45. $\sqrt[9]{72}$
46. $\sqrt[8]{63}$
47. $\sqrt[4]{0.0025}$
48. $\sqrt[5]{0.00032}$
49. $\sqrt[4]{3600}$
50. $\sqrt[6]{4500}$
51. $\sqrt[5]{35.4}$
52. $\sqrt[10]{1.8}$

Simplify each algebraic expression. Assume that all variables represent positive real numbers. Leave the answers in exponential notation. See Example 3.

53. $\left(2x^{\frac{1}{3}}\right)^{3}$

54. $\left(3x^{\frac{1}{2}}\right)^{4}$

55. $\left(9a^{4}\right)^{-\frac{1}{2}}$

56. $\left(16a^{3}\right)^{-\frac{1}{4}}$

57. $8x^{2}\cdot x^{\frac{1}{2}}$

58. $3x^{3}\cdot x^{\frac{2}{3}}$

59. $5a^{2}\cdot a^{-\frac{1}{3}}\cdot a^{\frac{1}{2}}$

60. $a^{\frac{2}{3}}\cdot a^{-\frac{3}{5}}\cdot a^{0}$

61. $\dfrac{x^{\frac{3}{4}}}{x^{\frac{1}{6}}}$

62. $\dfrac{a^{\frac{2}{3}}}{a^{\frac{1}{9}}}$

63. $\dfrac{x^{\frac{2}{5}}}{x^{-\frac{1}{10}}}$

64. $\dfrac{a^{\frac{1}{2}}}{a^{-\frac{2}{3}}}$

65. $\dfrac{a^{\frac{3}{4}}\cdot a^{\frac{1}{8}}}{a^{2}}$

66. $\dfrac{x^{\frac{2}{3}}\cdot x^{\frac{4}{3}}}{x^{2}}$

67. $\dfrac{a^{\frac{1}{2}}\cdot a^{-\frac{3}{4}}}{a^{-\frac{1}{2}}}$

68. $\dfrac{x^{\frac{2}{3}}x^{-1}}{x^{-\frac{3}{2}}}$

69. $\dfrac{a^{\frac{3}{2}}b^{\frac{4}{5}}}{a^{-\frac{1}{2}}b^{2}}$

70. $\dfrac{a^{\frac{3}{4}}b^{-\frac{1}{3}}}{a^{\frac{3}{2}}b^{\frac{1}{6}}}$

71. $\left(2x^{\frac{1}{2}}y^{\frac{1}{3}}\right)^{3}$

72. $\left(a^{\frac{1}{2}}a^{\frac{1}{3}}\right)^{6}$

73. $\left(4x^{-\frac{3}{4}}y^{\frac{1}{5}}\right)^{-2}$

74. $\left(81a^{-8}b^{2}\right)^{-\frac{1}{4}}$

75. $\left(-x^{3}y^{6}z^{-6}\right)^{\frac{2}{3}}$

76. $\left(9x^{2}y^{-4}z^{-3}\right)^{\frac{3}{2}}$

77. $\left(\dfrac{x^{2}y^{-3}}{z^{4}}\right)^{-\frac{1}{2}}$

78. $\left(\dfrac{27a^{3}b^{6}}{c^{9}}\right)^{-\frac{1}{3}}$

79. $\left(\dfrac{16a^{-4}b^{3}}{c^{4}}\right)^{\frac{3}{4}}$

80. $\left(\dfrac{-27a^{2}b^{3}}{c^{-3}}\right)^{\frac{1}{3}}$

81. $\dfrac{\left(x^{\frac{1}{4}}y^{\frac{1}{2}}\right)^{3}}{x^{\frac{1}{2}}y^{\frac{1}{4}}}$

82. $\dfrac{\left(x^{\frac{1}{2}}y\right)^{-\frac{1}{3}}}{x^{\frac{2}{3}}y^{-1}}$

83. $\dfrac{\left(8x^{2}y\right)^{-\frac{1}{3}}}{\left(5x^{\frac{1}{3}}y^{-\frac{1}{2}}\right)^{2}}$

84. $\dfrac{\left(25a^{4}b^{-1}\right)^{\frac{1}{2}}}{\left(2a^{\frac{1}{5}}b^{\frac{3}{5}}\right)^{3}}$

85. $\left(\dfrac{a^{-3}b^{\frac{1}{3}}}{a^{\frac{1}{2}}b}\right)^{\frac{1}{2}} \cdot \left(\dfrac{ab^{\frac{1}{2}}}{a^{-\frac{2}{3}}b^{-1}}\right)^{\frac{1}{2}}$

86. $\left(\dfrac{x^2y^{\frac{1}{3}}}{x^{\frac{1}{2}}y^{\frac{3}{2}}}\right)^{\frac{1}{2}} \cdot \left(\dfrac{x^{-\frac{1}{2}}y^{\frac{2}{3}}}{x^{-1}y^{\frac{3}{4}}}\right)^{2}$

87. $\dfrac{\left(27xy^{\frac{1}{2}}\right)^{\frac{1}{3}}}{\left(25x^{-\frac{1}{2}}y\right)^{\frac{1}{2}}} \cdot \dfrac{\left(x^{\frac{1}{2}}y\right)^{\frac{1}{6}}}{\left(16x^{\frac{1}{3}}y\right)^{\frac{1}{2}}}$

88. $\dfrac{\left(4a^{-6}b\right)^{\frac{1}{2}}}{\left(7a^2b^3\right)^{-1}} \cdot \dfrac{\left(49a^4b^3\right)^{-\frac{1}{2}}}{\left(64a^{-3}b^6\right)^{\frac{2}{3}}}$

Simplify each expression by first changing it into an equivalent expression with rational exponents. Rewrite the answer in simplified radical form. Assume that all variables represent positive real numbers. See Example 4.

89. $\sqrt{x} \cdot \sqrt[3]{x}$

90. $\sqrt[3]{x^2} \cdot \sqrt[5]{x^3}$

91. $\dfrac{\sqrt[4]{y^3}}{\sqrt[6]{y}}$

92. $\dfrac{\sqrt[3]{x^4}}{\sqrt[4]{x}}$

93. $\dfrac{\sqrt[3]{x^2}\,\sqrt[5]{x^6}}{\sqrt{x^3}}$

94. $\dfrac{a\sqrt[4]{a}}{\sqrt[3]{a}\sqrt{a}}$

95. $\sqrt{\sqrt[3]{y}}$

96. $\sqrt[5]{\sqrt{x}}$

97. $\sqrt[3]{\sqrt[3]{x}}$

98. $\sqrt{\sqrt{a}}$

99. $\sqrt[15]{(7a)^5}$

100. $\sqrt[21]{(3x)^7}$

101. $\sqrt[4]{\sqrt[3]{\sqrt{x}}}$

102. $\sqrt[5]{\sqrt[4]{\sqrt[3]{x}}}$

103. $\left(\sqrt[3]{a^4bc^2}\right)^{15}$

104. $\left(\sqrt[4]{a^3b^6c}\right)^{12}$

Applications

Solve.

105. According to Kepler's Law of Periods, the square of the orbital period of a planet is directly proportional to the cube of the semi-major axis of its orbit, or the cube of the planet's maximum distance from the sun. Writing this relationship as an equation solved for the period of a planet gives us $p = d^{\frac{3}{2}}$ where p is the orbital period of a planet represented in Earth years and d is the planet's semi-major axis represented in astronomical units (AU). Using the given equation, if the planet Mercury has a semi-major axis of 0.39 AU, what is the orbital period of Mercury in Earth years? Round your answer to the nearest hundredth.

106. The orbital period of Saturn is about 29.5 Earth years.

a. Solving Kepler's Law of Periods (see Exercise 105) for the planet's maximum distance from the sun, gives the equation $d = p^{\frac{2}{3}}$. Use this equation to calculate the maximum distance away from the sun that Saturn reaches in astronomical units. Round your answer to the nearest hundredth.

b. Given that Earth's semi-major axis is 1 AU, about how many times further from the sun does Saturn travel than Earth?

107. The width of a rectangle is $\sqrt[3]{64^2}$ ft and the length is $216^{\frac{2}{3}}$ ft. What is the area of the rectangle?

108. The motion of a simple pendulum is represented by the following equation, where T = the pendulum period, L = length, and g = acceleration of gravity.

$$T = 2\pi\sqrt{\frac{L}{g}}$$

If the length of the pendulum is $320^{\frac{2}{3}}$ meters and the acceleration of gravity is equal to 9.8 meters per seconds squared, what is the period of the pendulum? Use $\pi = 3.14$ and round your answer to the nearest hundredth.

109. A crew of construction workers are disassembling the inside of a building and dropping things into dumpsters at the base of the building. The equation $v_a = \frac{(64d)^{\frac{1}{2}}}{2}$ is used to find the average velocity, or speed, in feet per second of an object that has fallen a distance of d feet.

a. What is the average velocity, to the nearest hundredth, of a lighting fixture that fell 25 feet?

b. What is the average velocity, to the nearest hundredth, of a ceiling tile that fell 80 feet?

110. Isaac Newton fell asleep under an apple tree thinking about math. While he was sleeping, a squirrel knocked an apple off of a branch of the tree. The equation $v = (19.8d)^{\frac{1}{2}}$ can be used to find the velocity v, in meters per second, of the apple after dropping a distance d, where d is in meters.

a. If the apple was connected to a branch 2 m above Newton's head, what was the velocity of the apple, to the nearest hundredth, when it hit Newton's head?

b. If the squirrel knocked a second apple off a branch that was 5 m above Newton's head, what was the velocity of the apple, to the nearest hundredth, when it hit Newton's head?

c. Suppose the second apple missed Newton's head and landed on the ground instead. If Newton's head was 0.8 m above the ground, what was the velocity of the apple, to the nearest hundredth, when it hit the ground?

111. An amusement park is creating signs to indicate the velocity of the roller coaster car on certain hills of the most popular rides. A roller coaster car gains kinetic energy as it goes down a hill. The velocity, or speed, of an object in kilometers per hour (kph) can be determined by $V = \left(\frac{2k}{m}\right)^{\frac{1}{2}}$, where k is the kinetic energy of the object in joules (J) and m is the mass of the object in kilograms (kg).

a. For the most popular roller coaster, the car has a mass of 300 kg and the car has a kinetic energy of 375,000 J on the first hill. What velocity does the car obtain on the first hill?

b. For the second most popular roller coaster, the car has a mass of 350 kg and the car has a kinetic energy of 70,000 on the first hill. What velocity does the car obtain on the first hill?

Writing & Thinking

112. Is $\sqrt[5]{a} \cdot \sqrt{a}$ the same as $\sqrt[5]{a^2}$? Explain why or why not.

113. Assume that x represents a positive real number. Describe what kind of number the exponent n must be for x^n to mean

a. a product.

b. a quotient.

c. 1.

d. a radical expression.