Margin Exercise Answers
1. $y = 4$ **2.** 15 cm **3.** $y = 2$ **4.** 173 pounds **5.** $z = 648$ **6.** 400 feet **7.** 200 cubic inches **8.** 8000 pounds

8.8 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. If two variables are inversely proportional, an increase in the value of one variable must be accompanied by a/an _____ in the other.
2. If a variable varies (directly or inversely) with more than one other variable, this variation is said to be a/an _____ variation.
3. When two variables vary directly, an increase in one variable indicates a/an _____ in the other.
4. When two variables vary so that their product is constant, the two variables vary _____.
5. If there is a combined variation that is all direct variation, it is a/an _____ variation.
6. The letter k often represents the constant of _____.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. The number of hamburgers eaten varies inversely with calories consumed.
8. The equation $y = \frac{k}{x}$ represents direct variation.
9. Distance and time varies directly, which means they are directly proportional.
10. The circumference of a circle varies directly with its radius.

Practice

Use the information given to find the unknown value. See Examples 1, 3, and 5.

1. If y varies directly as x, and $y = 3$ when $x = 9$, find y if $x = 7$.
2. If y is directly proportional to x^2, and $y = 3$ when $x = 2$, what is y when $x = 8$?
3. If y varies inversely as x, and $y = 5$ when $x = 8$, find y if $x = 20$.
4. If y is inversely proportional to x, and $y = 5$ when $x = 4$, what is y when $x = 2$?
5. If y varies inversely as x^2, and $y = -8$ when $x = 2$, find y if $x = 3$.
6. If y is inversely proportional to x^3, and $y = 40$ when $x = \frac{1}{2}$, what is y when $x = \frac{1}{3}$?
7. If y is directly proportional to the square root of x, and $y = 6$ when $x = \frac{1}{4}$, what is y when $x = 9$?

8. If y is directly proportional to the square of x, and $y = 80$ when $x = 4$, what is y when $x = 6$?

9. z varies jointly as x and y, and $z = 60$ when $x = 2$ and $y = 3$. Find z if $x = 3$ and $y = 4$.

10. z varies jointly as x and y, and $z = -6$ when $x = 5$ and $y = 8$. Find z if $x = 12$ and $y = 15$.

11. z varies jointly as x and y^2, and $z = 63$ when $x = 5$ and $y = 3$. Find z if $x = \frac{10}{3}$ and $y = 2$.

12. z varies jointly as x^2 and y, and $z = 20$ when $x = 2$ and $y = 3$. Find z if $x = 4$ and $y = \frac{7}{10}$.

13. z varies directly as x and inversely as y^2. If $z = 5$ when $x = 1$ and $y = 2$, find z if $x = 2$ and $y = 1$.

14. z varies directly as x^3 and inversely as y^2. If $z = 24$ when $x = 2$ and $y = 2$, find z if $x = 3$ and $y = 2$.

15. z varies directly as $\sqrt{x}$ and inversely as y. If $z = 24$ when $x = 4$ and $y = 3$, find z if $x = 9$ and $y = 2$.

16. z varies directly as x^2 and inversely as $\sqrt{y}$. If $z = 108$ when $x = 6$ and $y = 4$, find z if $x = 4$ and $y = 9$.

17. s varies directly as the sum of r and t and inversely as w. If $s = 24$ when $r = 7$ and $t = 8$ and $w = 9$, find s if $r = 9$ and $t = 3$ and $w = 18$.

18. s varies directly as r and inversely as the difference of t and u. If $s = 36$ when $r = 12$ and $t = 9$ and $u = 6$, find s if $r = 18$ and $t = 11$ and $u = 8$.

19. L varies jointly as m and n and inversely as p. If $L = 6$ when $m = 7$ and $n = 8$ and $p = 12$, find L if $m = 15$ and $n = 14$ and $p = 10$.

20. W varies jointly as x and y and inversely as z. If $W = 10$ when $x = 6$ and $y = 5$ and $z = 2$, find W if $x = 12$ and $y = 6$ and $z = 3$.

Applications

Solve.

21. The distance a free-falling object falls is directly proportional to the square of the time it falls (before it hits the ground). If an object fell 256 feet in 4 seconds, how far would it have fallen by the end of 5 seconds?

22. The length a hanging spring stretches varies directly with the weight placed on the end. If a spring stretches 5 in. with a weight of 10 lb, how far will the spring stretch if the weight is increased to 12 lb?

23. The total price (P) of gasoline purchased varies directly as the number of gallons purchased. If 10 gallons are purchased for $23.40, what will be the price of 15 gallons?

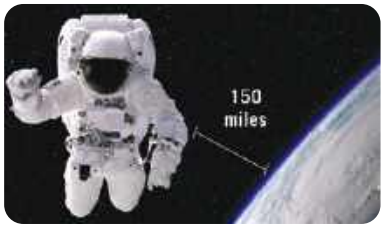
24. Research shows that the value of gold and the value of the dollar are inversely proportional. In 2016, gold cost $1200 per ounce and the dollar had a rating of 93 on the US dollar index. In 2017, the cost of gold was $1300 per ounce. What was the 2017 rating of the dollar? (Round your answer to the nearest hundredth.)

25. The circumference of a circle varies directly as the diameter. A circular pizza pie with a diameter of 1 foot has a circumference of 3.14 feet. What will be the circumference of a pizza pie with a diameter of 1.5 feet?

26. The area of a circle varies directly as the square of its radius. A circular pizza pie with a radius of 6 in. has an area of 113.04 in.^{2} What will be the area of a pizza pie with a radius of 9 in.?

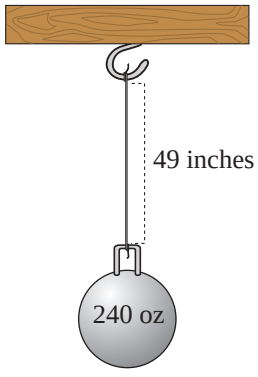
27. Several triangles have the same area. In this set of triangles, the height and base are inversely proportional. In one such triangle, the height is 5 m and the base is 12 m. Find the height of the triangle in this set with a base of 10 m.

28. If an astronaut weighs 250 pounds on the surface of the earth, what will the astronaut weigh 150 miles above the earth? Assume that the radius of the earth is 4000 miles, and round to the nearest tenth. (See Example 4.)



29. The elongation (E) in a wire when a mass (m) is hung at its free end varies jointly as the attached mass and the length (l) of the wire and inversely as the cross-sectional area (A) of the wire. The elongation is 0.0055 cm when a mass of 120 g is attached to a wire 330 cm long with a cross-sectional area of 0.4 cm^2. Find the elongation if a mass of 160 g is attached to the same wire.

30. When a mass of 240 oz is suspended by a wire 49 in. long whose cross-sectional area is 0.035 in.^{2}, the elongation of the wire is 0.016 in. Find the elongation if the same mass is suspended by a 28 in. wire of the same material with a cross-sectional area of 0.04 in.^{2} (See Exercise 29.)



31. The safe load (L) of a wooden beam supported at both ends varies jointly as the width (w) and the square of the depth (d) and inversely as the length (l). A beam 4 in. wide, 6 in. deep, and 12 ft long supports a load of 4800 lb safely. What is the safe load of a beam of the same material that is 6 in. wide, 10 in. deep, and 15 ft long?

32. A wooden beam 2 in. wide, 8 in. deep, and 14 ft long holds up to 2400 lb. What load would a beam 3 in. wide, 6 in. deep, and 15 ft long, of the same material, support? (See Exercise 31.)

33. The gravitational force of attraction (F) between two bodies varies directly as the product of their masses (m_1 and m_2) and inversely as the square of the distance (d) between them. The gravitational force between a 5-kg mass and a 2-kg mass 1 m apart is 1.5×10^{-10} N. Find the force between a 24-kg mass and a 9-kg mass that are 6 m apart. (N represents a unit of force called a Newton.)

34. In Exercise 33, what is the force if the distance between the 24 kg mass and the 9 kg mass is cut in half?

35. The total price (P) of gasoline purchased varies directly as the number of gallons purchased. If 10 gallons are purchased for \$39.80, what will be the price of 15 gallons?

36. The distance that an object falls is directly proportional to the square of the time that has passed since the object started to fall. A rock falls a distance of 64 feet in 2 seconds. How long will it take the rock to fall a distance of 100 feet?

37. For a certain type of wooden beam that carries a load at its center, the safe load (SL) varies jointly as the width w and the cube of the depth (d) and inversely as the square of the length (l). A wooden beam that is 4 inches wide, 6 inches deep, and 12 feet long can safely support a load of 2400 pounds.

a. Set up the variation equation.

b. Determine the constant of variation.

c. How much weight can a wooden beam that is 5 inches wide, 6 inches deep, and 10 feet long safely support?

Solve the following lifting force problems.

Lifting Force

The lifting force (or lift) (L) in pounds exerted by the atmosphere on the wings of an airplane is related to the area (A) of the wings in square feet and the speed (or velocity) (v) of the plane in miles per hour by the formula $L = kAv^2$, where k is the constant of variation.

38. If the lift is 9600 lb for a wing area of 120 ft^2 at a speed of 80 mph, find the lift of the same wing at a speed of 100 mph.

39. The lift for a wing of area 280 ft^2 is 34,300 lb when the plane is traveling at 210 mph. What is the lift if the speed is decreased to 180 mph?

40. The lift for a wing with an area of 144 ft^2 is 10,000 lb when the plane is traveling at 150 mph. What is the lift if the speed is decreased to 120 mph?

41. A plane traveling 140 mph with wing area 195 ft^2 has 12,500 lb of lift exerted on the wings. Find the lift for the same plane traveling at 168 mph.

Solve the following pressure problems.

Pressure

Boyle's Law states that if the temperature of a gas sample remains the same, the pressure (P) of the gas is related to the volume (V) by the formula $P = \frac{k}{V}$, where k is the constant of variation.

42. A pressure of 1600 lb per ft^2 is exerted by 2 ft^3 of air in a cylinder. If a piston is pushed into the cylinder until the pressure is 1800 lb per ft^2, what will be the volume of the air? Round to the nearest tenth.

43. The volume of gas in a container is 300 cm^3 when the pressure on the gas is 20 g per cm^2. What will be the volume if the pressure is increased to 30 g per cm^2?

44. The pressure in a canister of gas is 1360 g per in.^{2} when the volume of gas is 5 in.^{3} If the volume is reduced to 4 in.^{3}, what is the pressure?

45. A scuba diver is using a diving tank that can hold 6 liters of air. If the tank has a pressure rating of 220 bar when full, what is the pressure rating when the volume of gas is 4 liters?

Solve the following electricity problems.

Electricity

The resistance (R) (in ohms), in a wire is given by the formula $R = \frac{kL}{d^2}$, where k is the constant of variation, L is the length of the wire and d is the diameter.

46. The resistance of a wire 500 ft long with a diameter of 0.01 in. is 20 ohms. What is the resistance of a wire 1500 ft long with a diameter of 0.02 in.?

47. The resistance is 2.6 ohms when the diameter of a wire is 0.02 in. and the wire is 10 ft long. Find the resistance of the same type of wire with a diameter of 0.01 in. and a length of 5 ft.

48. Tristan's car stereo uses a 5-ft audio wire with diameter 0.025 in. and resistance of 1.6 ohms. What is the resistance of 8 ft of the same type of audio wire?

49. Nicole purchased a spool of wire with diameter 0.01 in. for the speakers in her home audio system. If the resistance of 15 ft of this wire is 6 ohms, what is the resistance of 25 ft of the wire?

Solve the following lever problems.

Levers

If a lever is balanced with weight on opposite sides of its balance point, then the following proportion exists:

$$\frac{W_1}{W_2} = \frac{L_2}{L_1} \text{ or } W_1L_1 = W_2L_2$$

where $L_1 + L_2 = L,$ the total length of the lever.

50. How much weight can be raised at one end of a bar 8 ft long by the downward force of 60 lb when the balance point is $\frac{1}{2}$ ft from the unknown weight?

51. Where should the balance point of a bar 12 ft long be located if a 120-lb force is to raise a load weighing 960 lb?

52. Find the location of the balance point of a 25-ft board that can raise a 300-lb package with a downward force of 75 lb.

53. How much weight can be raised on one end of a 17-meter board by 90 kilograms, if the balance point is 5 meters from the unknown weight?

Writing & Thinking

54. Explain, in your own words, the meaning of the following terms.

a. Direct variation

b. Joint variation

c. Inverse variation

d. Combined variation

Discuss an example of each type of variation that you have observed in your daily life.