

Solution

$$\begin{aligned} \text{a. } x^3 - 8 &= x^3 - 2^3 \\ &= (x-2)(x^2 + 2 \cdot x + 2^2) \\ &= (x-2)(x^2 + 2x + 4) \end{aligned}$$

Note: Remember that the second polynomial is not a perfect square trinomial and cannot be factored.

$$\begin{aligned} \text{b. } x^6 + 64y^3 &= (x^2)^3 + (4y)^3 \\ &= (x^2 + 4y)\left[(x^2)^2 - 4y \cdot x^2 + (4y)^2\right] \\ &= (x^2 + 4y)(x^4 - 4x^2y + 16y^2) \end{aligned}$$

- c. Factor out the GCF first. Then factor the **difference of two cubes**.

$$\begin{aligned} 16y^{12} - 250 &= 2(8y^{12} - 125) \\ &= 2\left[(2y^4)^3 - 5^3\right] \\ &= 2(2y^4 - 5)\left[(2y^4)^2 + (5)(2y^4) + 5^2\right] \\ &= 2(2y^4 - 5)(4y^8 + 10y^4 + 25) \end{aligned}$$

Now work margin exercise 4.**Margin Exercise Answers**

1. a. $7a(x-7)(x+7)$ b. $(y^3+10)(y^3-10)$ 2. a. Not factorable b. $5(9x^2+4)$
 3. a. $(z+20)^2$ b. $(y-7)^2$ c. $3z(x-3y)^2$ d. $(y+4-z)(y+4+z)$
 4. a. $(y-3)(y^2+3y+9)$ b. $(2y-x^2)(4y^2+2x^2y+x^4)$ c. $6(2x^4-5)(4x^8+10x^4+25)$

7.4 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

- Factoring a perfect square trinomial gives a square _____.
- In a perfect square trinomial, both the first and last terms must be perfect _____.
- If the first term of a perfect square trinomial is x^2 , and the last term is of the form a^2 , then the middle term must be of the form _____ or _____.
- The formula for factoring the difference of cubes is $x^3 - a^3 =$ _____.
- The formula for factoring the sum of two cubes is $x^3 + a^3 =$ _____.
- The first 6 perfect cubes are 1, 8, _____, _____, _____, and _____.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. The expression $x^2 + 20x + 100$ is a perfect square trinomial.
8. When factoring polynomials, always look for a common monomial factor first.
9. The sum of two squares, $(x^2 + a^2)$, is factorable.
10. Sixty-four is a perfect square and a perfect cube.

Practice

Completely factor each of the given polynomials. If a polynomial cannot be factored, write "not factorable." See Examples 1 through 4.

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|-----------------------|------------------------------|-----------------------|
| 1. $x^2 - 25$ | 22. $9y^2 + 12y + 4$ | 43. $4x^3 - 32$ |
| 2. $y^2 - 121$ | 23. $16x^2 - 40x + 25$ | 44. $64x^3 + 27y^3$ |
| 3. $81 - y^2$ | 24. $9x^2 - 12x + 4$ | 45. $54x^3 - 2y^3$ |
| 4. $25 - z^2$ | 25. $4x^3 - 64x$ | 46. $3x^4 + 375xy^3$ |
| 5. $2x^2 - 128$ | 26. $50x^3 - 8x$ | 47. $x^3y + y^4$ |
| 6. $3x^2 - 147$ | 27. $2x^3y + 32x^2y + 128xy$ | 48. $x^4y^3 - x$ |
| 7. $4x^4 - 64$ | 28. $3x^2y - 30xy + 75y$ | 49. $x^2y^2 - x^2y^5$ |
| 8. $5x^4 - 125$ | 29. $y^2 + 6y + 9$ | 50. $2x^2 - 16x^2y^3$ |
| 9. $y^2 + 100$ | 30. $y^2 + 4y + 4$ | 51. $24x^4y + 81xy^4$ |
| 10. $4x^2 + 49$ | 31. $x^2 - 20x + 100$ | 52. $x^6 - 64y^3$ |
| 11. $y^2 - 16y + 64$ | 32. $25x^2 - 10x + 1$ | 53. $x^6 - y^9$ |
| 12. $z^2 + 18z + 81$ | 33. $x^4 + 10x^2y + 25y^2$ | 54. $64x^2 + 1$ |
| 13. $-4x^2 + 100$ | 34. $16x^4 + 8x^2y + y^2$ | 55. $27x^3 + y^6$ |
| 14. $-12x^4 + 3$ | 35. $x^3 - 125$ | 56. $x^3 + 64z^3$ |
| 15. $9x^2 - 25$ | 36. $x^3 - 64$ | 57. $8x^3 + y^3$ |
| 16. $4x^2 - 49$ | 37. $y^3 + 216$ | 58. $x^3 + 125y^3$ |
| 17. $y^2 - 10y + 25$ | 38. $y^3 + 1$ | 59. $8y^3 - 8$ |
| 18. $x^2 + 12x + 36$ | 39. $x^3 + 27y^3$ | 60. $36x^3 + 36$ |
| 19. $4x^2 - 4x + 1$ | 40. $8x^3 + 1$ | 61. $9x^2 - y^2$ |
| 20. $49x^2 - 14x + 1$ | 41. $x^2 + 64y^2$ | 62. $x^2 - 4y^2$ |
| 21. $25x^2 + 30x + 9$ | 42. $3x^3 + 81$ | 63. $x^4 - 16y^4$ |

74. Compound interest is interest earned on interest. If a principal P is invested and compounded annually (once a year) at a rate of r , then the amount, A_1 accumulated in one year is $A_1 = P + Pr$.

In factored form, we have $A_1 = P + Pr = P(1 + r)$.

At the end of the second year the amount accumulated is $A_2 = (P + Pr) + (P + Pr)r$.

- a. Write the expression for A_2 in factored form similar to that for A_1 .
- b. Write an expression for the amount accumulated in three years, A_3 , in factored form.
- c. Write an expression for A_n the amount accumulated in n years.
- d. Use the formula you developed in part c. and your calculator to find the amount accumulated if \$10,000 is invested at 6% and compounded annually for 20 years.

75. You may have heard of (or studied) the following rules for division of an integer by 3 and 9:

1. An integer is divisible by 3 if the sum of its digits is divisible by 3.
2. An integer is divisible by 9 if the sum of its digits is divisible by 9.

The proofs of both **1.** and **2.** can be started as follows.

Let abc represent a three-digit integer.

$$\begin{aligned}\text{Then } abc &= 100a + 10b + c \\ &= (99 + 1)a + (9 + 1)b + c \\ &= (\text{now you finish the proofs})\end{aligned}$$

Use the pattern just shown and prove both **1.** and **2.** for a four-digit integer.