

A complete summary of the rules for exponents includes the following eight rules.

### Summary of the Rules for Exponents

The following rules are true for any nonzero real numbers  $a$  and  $b$  and integers  $m$  and  $n$ .

1. The exponent 1:  $a = a^1$
2. The exponent 0:  $a^0 = 1$
3. The product rule:  $a^m \cdot a^n = a^{m+n}$
4. The quotient rule:  $\frac{a^m}{a^n} = a^{m-n}$
5. Negative exponents:  $a^{-n} = \frac{1}{a^n}$ .
6. Power rule:  $(a^m)^n = a^{mn}$ .
7. Power of a product:  $(ab)^n = a^n b^n$ .
8. Power of a quotient:  $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ .

PROPERTIES

#### Margin Exercise Answers

1. a.  $x^{15}$  b.  $\frac{1}{x^{12}}$  c.  $\frac{1}{y^{15}}$  d.  $\frac{1}{3^6}$  or  $\frac{1}{9^3}$  or  $\frac{1}{729}$  2. a.  $16x^2$  b.  $x^7 y^7$  c.  $81a^2 b^2$  d.  $\frac{1}{a^3 b^3}$   
 e.  $\frac{x^6}{y^8}$  3. a.  $\frac{x^7}{y^7}$  b.  $\frac{25}{36}$  c.  $\frac{27}{a^3}$  d.  $\frac{x^3}{216}$  4. a.  $\frac{-27x^3}{y^9}$  b.  $\frac{16b^2}{a^2}$  5.  $\frac{y^{15}}{x^{30}}$  6.  $\frac{64}{225x^{18}y^2}$

## 6.2 Exercises

### Concept Check

**Fill-in-the-Blank.** Complete each sentence using information found in this section.

1. Moving any term from the numerator to the denominator, or vice versa, changes the sign of the corresponding \_\_\_\_\_.
2. A power of a quotient (in fraction form) is found by raising both the \_\_\_\_\_ and the \_\_\_\_\_ to that power.
3. To find the value of a power raised to a power, \_\_\_\_\_ the exponents and \_\_\_\_\_ the base.
4. A power of a product can be found by \_\_\_\_\_ each factor to that power.
5. In an expression such as  $-x^2$ , we know that  $-1$  is understood to be the \_\_\_\_\_ of  $x^2$ .

**True/False.** Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

6. Taking the reciprocal of a fraction changes the sign of any exponent in the fraction.
7. For an exponent to refer to  $-7$  as the base,  $-7$  must be in parentheses.
8. When simplifying an expression with exponents, the rules for exponents must be used in a specific order or the answer will vary.
9. The expression  $-8^2$  simplifies to  $-64$ .

## Practice

Use the rules for exponents to simplify each of the expressions. Assume that all variables represent nonzero real numbers. See Examples 1 through 6.

- |                     |                                      |   |
|---------------------|--------------------------------------|---|
| 1. $-3^4$           | 20. $(-3x^4)^2$                      | 35. $\left(\frac{6m^3}{n^5}\right)^0$       |
| 2. $-5^2$           | 21. $4(-3x^2)^3$                     | 36. $\left(\frac{3x^2}{y^3}\right)^2$       |
| 3. $-2^4$           | 22. $7(2y^{-2})^4$                   | 37. $\left(\frac{-2x^2}{y^{-2}}\right)^2$   |
| 4. $-20^2$          | 23. $5(x^2y^{-1})$                   | 38. $\left(\frac{2x}{y^5}\right)^{-2}$      |
| 5. $(-10)^6$        | 24. $-3(7xy^2)^0$                    | 39. $\left(\frac{x}{y}\right)^{-2}$         |
| 6. $(-4)^6$         | 25. $-2(3x^5y^{-2})^{-3}$            | 40. $\left(\frac{2a}{b}\right)^{-1}$        |
| 7. $(a^3)^2$        | 26. $-4(5x^{-3}y)^{-1}$              | 41. $\left(\frac{3x}{y^{-2}}\right)^{-1}$   |
| 8. $(b^2)^{-4}$     | 27. $\left(\frac{a}{b}\right)^4$     | 42. $\left(\frac{4a^2}{b^{-3}}\right)^{-3}$ |
| 9. $(x^{-5})^2$     | 28. $\left(\frac{x}{2}\right)^3$     | 43. $\left(\frac{-3}{xy^2}\right)^{-3}$     |
| 10. $(x^{-2})^{-3}$ | 29. $\left(\frac{2}{3}\right)^2$     | 44. $\left(\frac{5xy^3}{y}\right)^2$        |
| 11. $(2^4)^{-2}$    | 30. $\left(\frac{a}{4}\right)^3$     | 45. $\left(\frac{m^2n^3}{mn}\right)^2$      |
| 12. $(2^{-3})^{-2}$ | 31. $\left(\frac{x}{y}\right)^6$     |   |
| 13. $(3y)^2$        | 32. $\left(\frac{2}{5}\right)^2$     |   |
| 14. $(ab)^4$        | 33. $\left(\frac{3x}{y}\right)^3$    |   |
| 15. $(-4xy)^2$      | 34. $\left(\frac{-4x}{y^2}\right)^2$ |   |
| 16. $(3x^{-2})^2$   |                                      |   |
| 17. $(xy)^{-6}$     |                                      |   |
| 18. $(a^3b^{-2})^3$ |                                      |   |
| 19. $(6x^3)^2$      |                                      |   |

46.  $\left(\frac{2ab^3}{b^2}\right)^4$

47.  $\left(\frac{-7^2x^2y}{y^3}\right)^{-1}$

48.  $\left(\frac{2ab^4}{b^2}\right)^{-3}$

49.  $\left(\frac{5x^3y}{y^2}\right)^2$

50.  $\left(\frac{2x^2y}{y^3}\right)^{-4}$

51.  $\left(\frac{x^3y^{-1}}{y^2}\right)^2$

52.  $\left(\frac{2a^2b^{-1}}{b^2}\right)^3$

53.  $\left(\frac{6y^5}{x^2y^{-2}}\right)^2$

54.  $\left(\frac{3x^4}{x^{-2}y^{-4}}\right)^3$

55.  $\frac{(7x^{-2}y)^2}{(xy^{-1})^2}$

56.  $\frac{(-5x^3y^4)^2}{(3x^{-3}y)^2}$

57.  $\frac{(3x^2y^{-1})^{-2}}{(6x^{-1}y)^{-3}}$

58.  $\frac{(2x^{-3})^{-3}}{(5y^{-2})^{-2}}$

59.  $\frac{(4x^{-2})(6x^5)}{(9y)(2y^{-1})}$


60.  $\frac{(5x^2)(3x^{-1})^2}{(25y^3)(6y^{-2})}$

61.  $\left(\frac{3xy^3}{4x^2y^{-3}}\right)^{-1}\left(\frac{2x^3y^{-1}}{9x^{-3}y^{-1}}\right)^2$

62.  $\left(\frac{5a^4b^{-2}}{6a^{-4}b^3}\right)^{-2}\left(\frac{5a^3b^4}{2^{-2}a^{-2}b^{-2}}\right)^3$

63.  $\left(\frac{6x^{-4}yz^{-2}}{4^{-1}x^{-4}y^3z^{-2}}\right)^{-1}\left(\frac{2^{-2}xyz^{-3}}{12x^2y^2z^{-1}}\right)^{-2}$

64.  $\left(\frac{3^{-5}a^5b^3c^{-1}}{3^{-2}abc}\right)^{-2}\left(\frac{7^{-1}a^{-4}bc^2}{7^{-2}a^{-3}bc^{-2}}\right)^{-2}$

 Use a graphing calculator to evaluate each expression. Round quotients to the nearest ten-thousandth, if necessary.

65.  $(2.1^2)^2$

66.  $(1.4^{-2})^5$

67.  $(3.8x)^4$

68.  $(5.2x^2)^3$

69.  $\left(\frac{8.1}{1.7}\right)^2$

70.  $\left(\frac{2.3}{4.5}\right)^3$